Quantum Algorithms for Beginners

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Quantum Information for Developers
ETH Zurich
2018

Quantum Algorithms

- There are a ***lot*** of quantum algorithms
  - The ‘Quantum Algorithm Zoo’ cites 392 papers on quantum algorithms
- Mostly, they solve specific mathematical problems
  - E.g. Factoring, matrix inversion
- Often cleverly combine smaller quantum sub-routines

This lecture

- Will focus on a few important algorithms / sub-routines:
  - Grover's search, phase estimation, factoring, matrix inversion (HHL), Hamiltonian simulation
- Mostly give high-level overviews
  - Hopefully enough detail to be able to implement Grover's search (and understand what's happening!)
- For a more complete introduction:
  - Ashley Montanaro's lecture notes www.people.maths.bris.ac.uk/~csxam/teaching
  - Ronald de Wolf's lecture notes www.homepages.cwi.nl/~rdewolf/qcnotes.pdf
  - *Quantum Computation and Quantum Information* by Nielsen and Chuang

We might divide quantum algorithms into two categories:

Polynomial speedup	Exponential speedup
Grover's Search	**Integer Factoring**
Quantum walks	**Matrix Inversion**
Graph algorithms	**Phase Estimation**
Minimum finding	Quantum Fourier Transform
	Quantum Simulation

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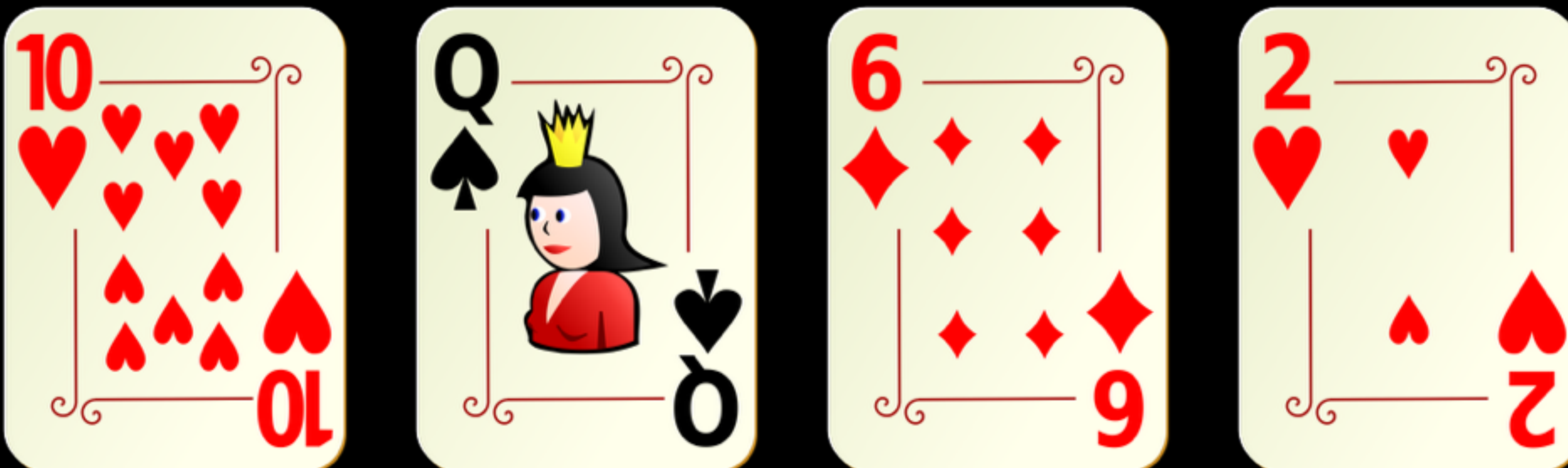

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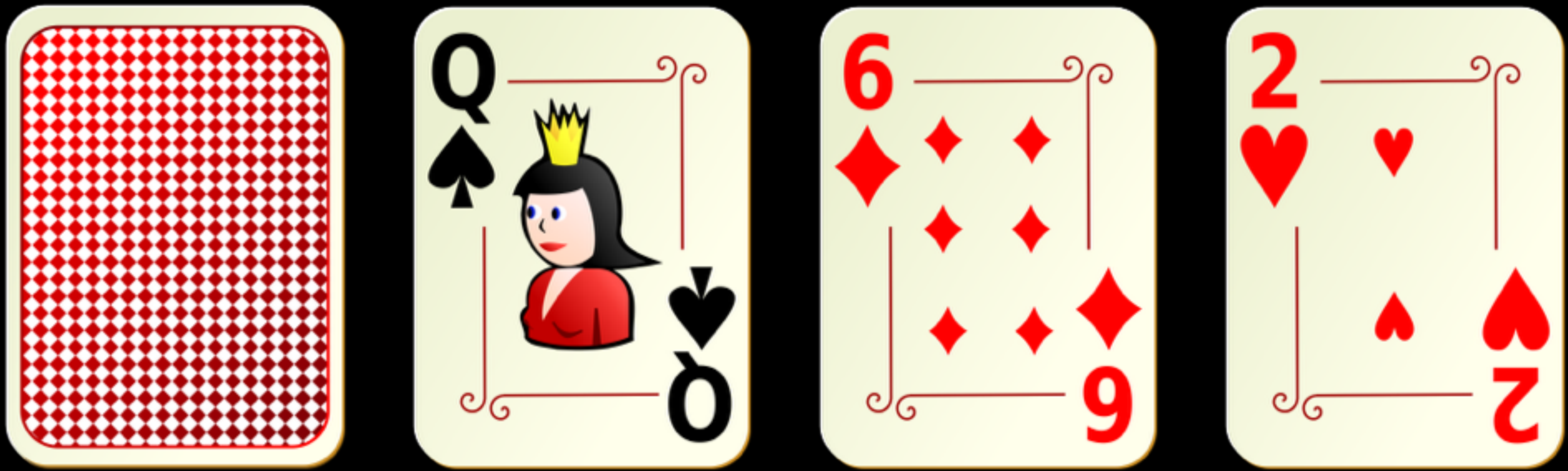

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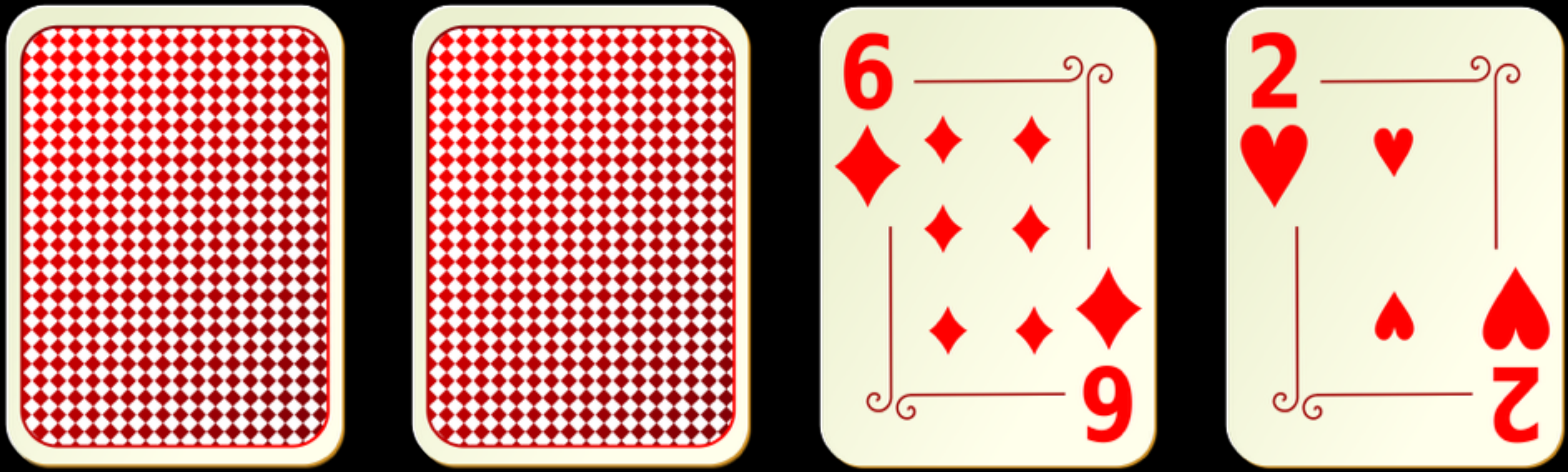

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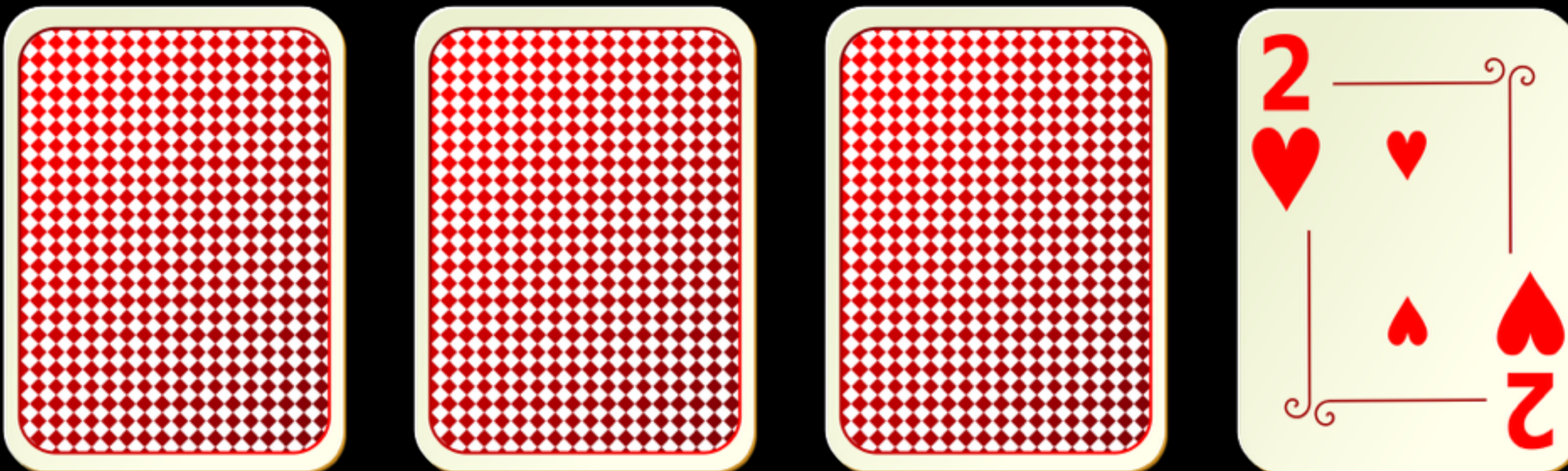

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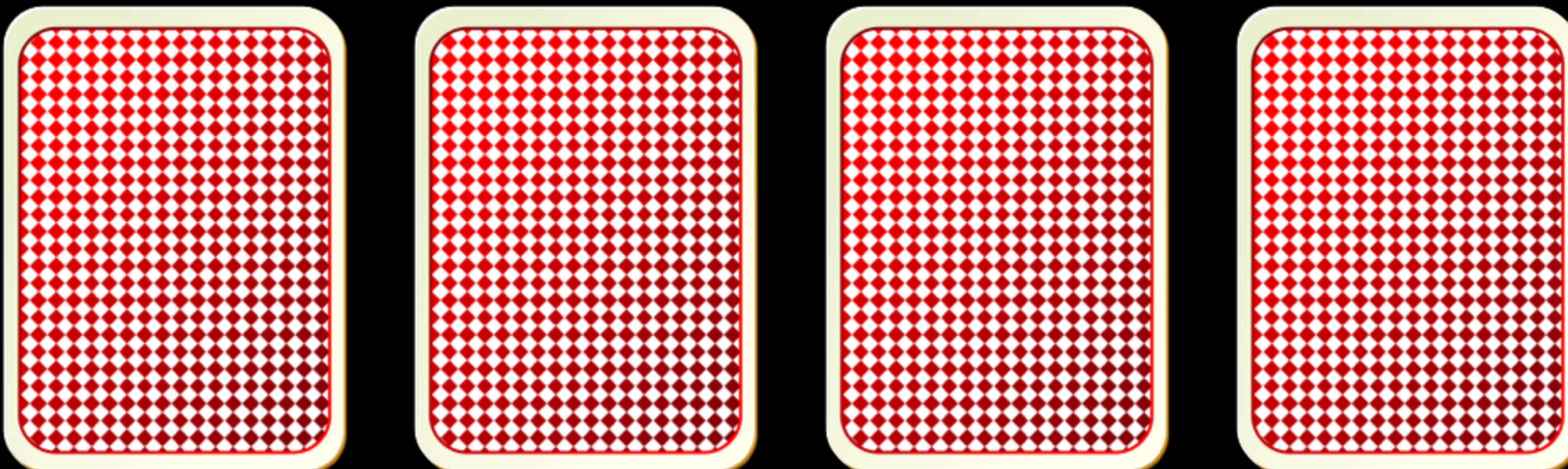

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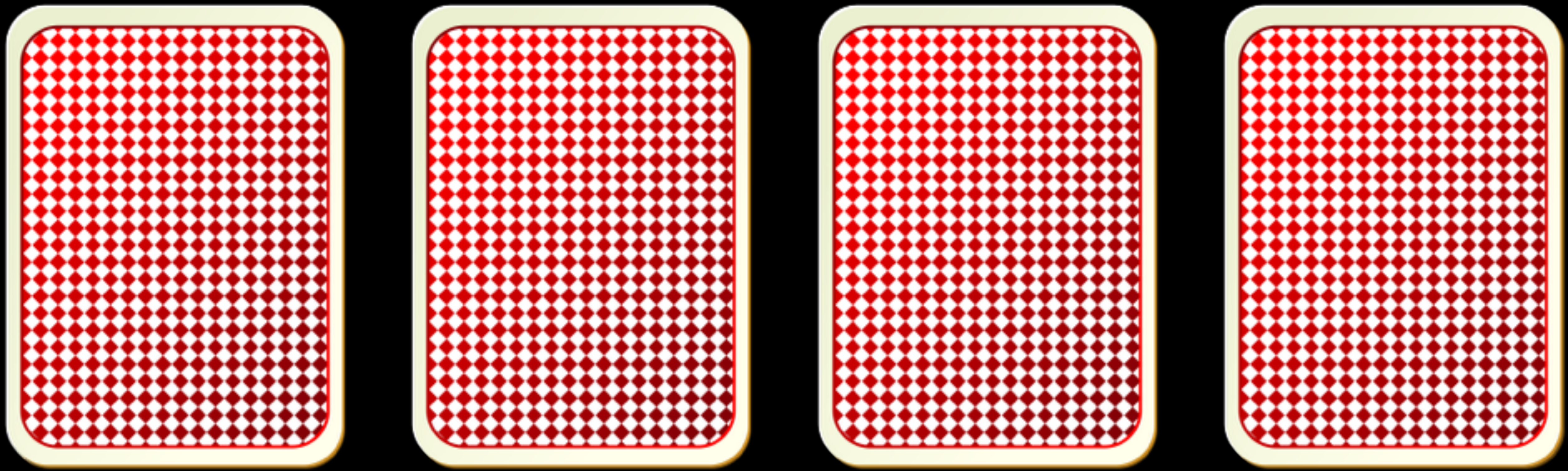

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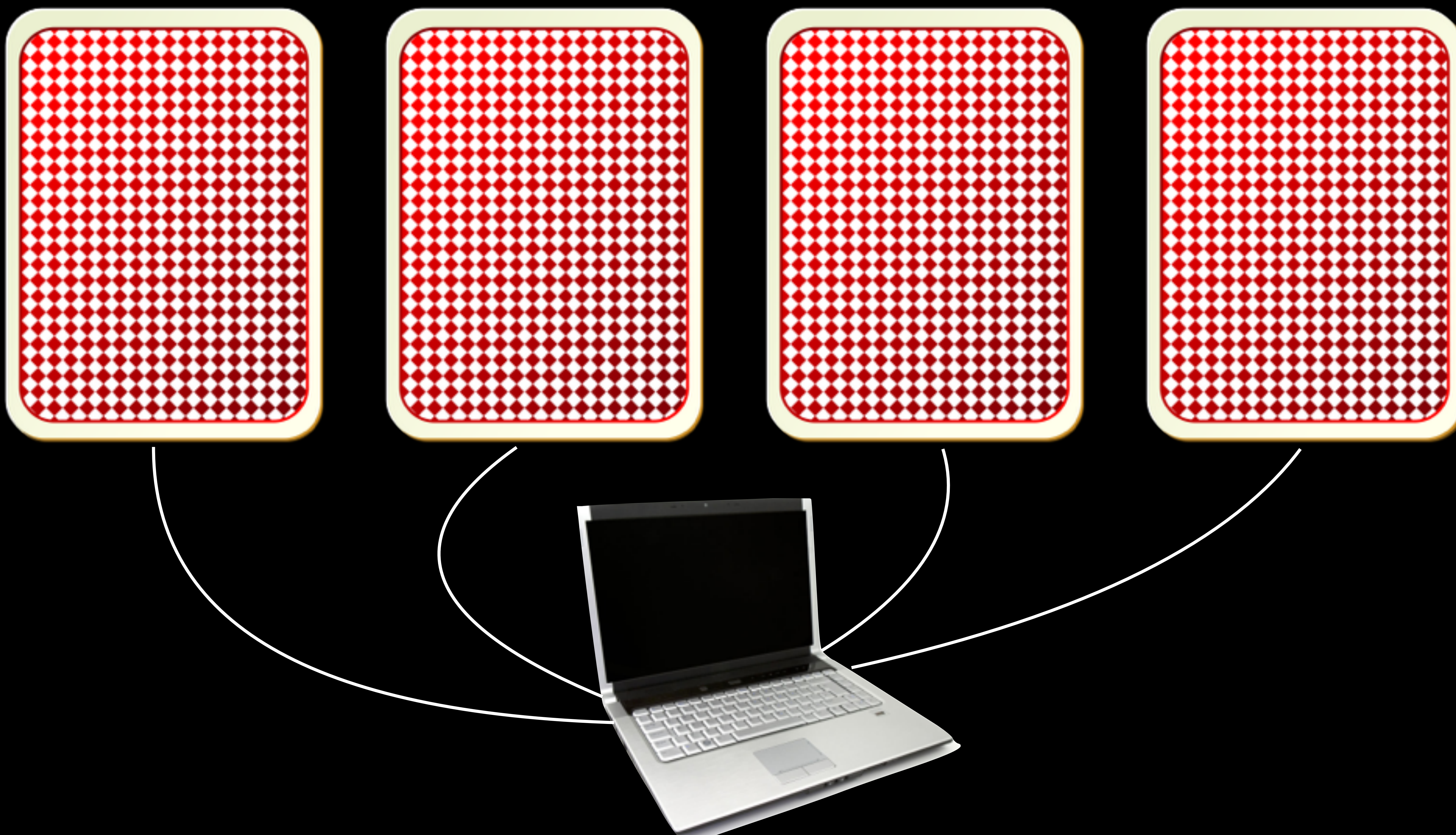

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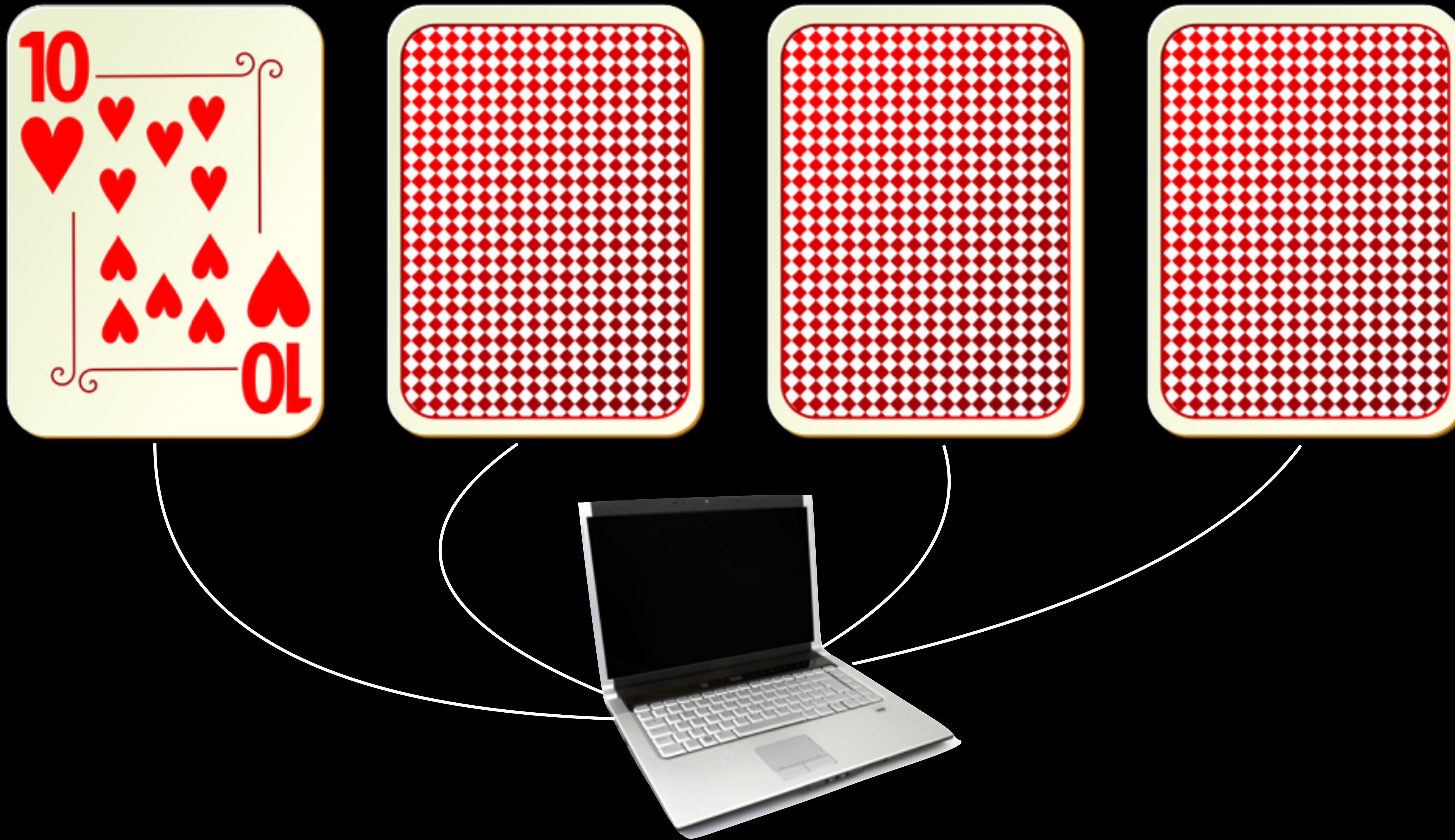

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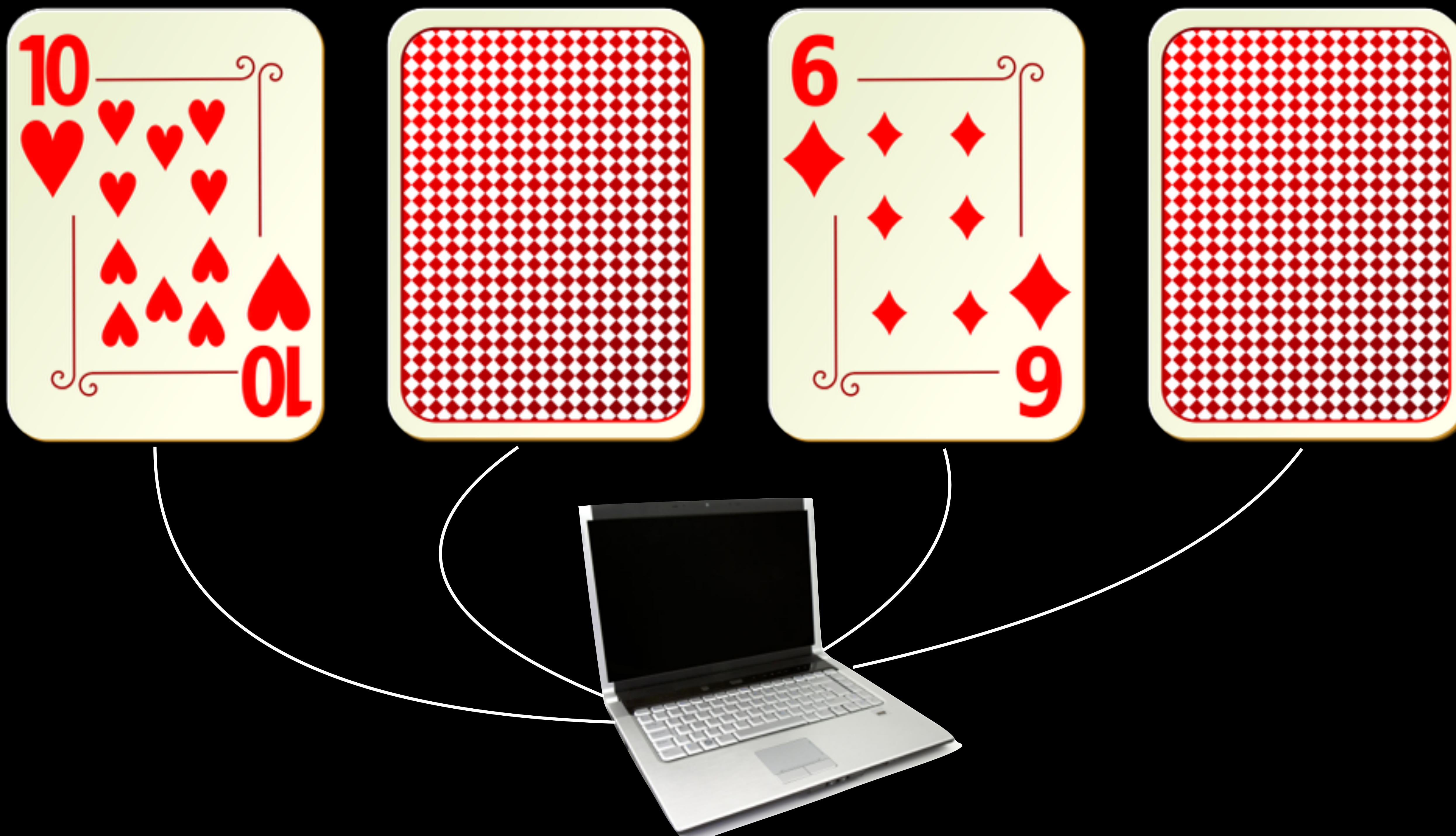

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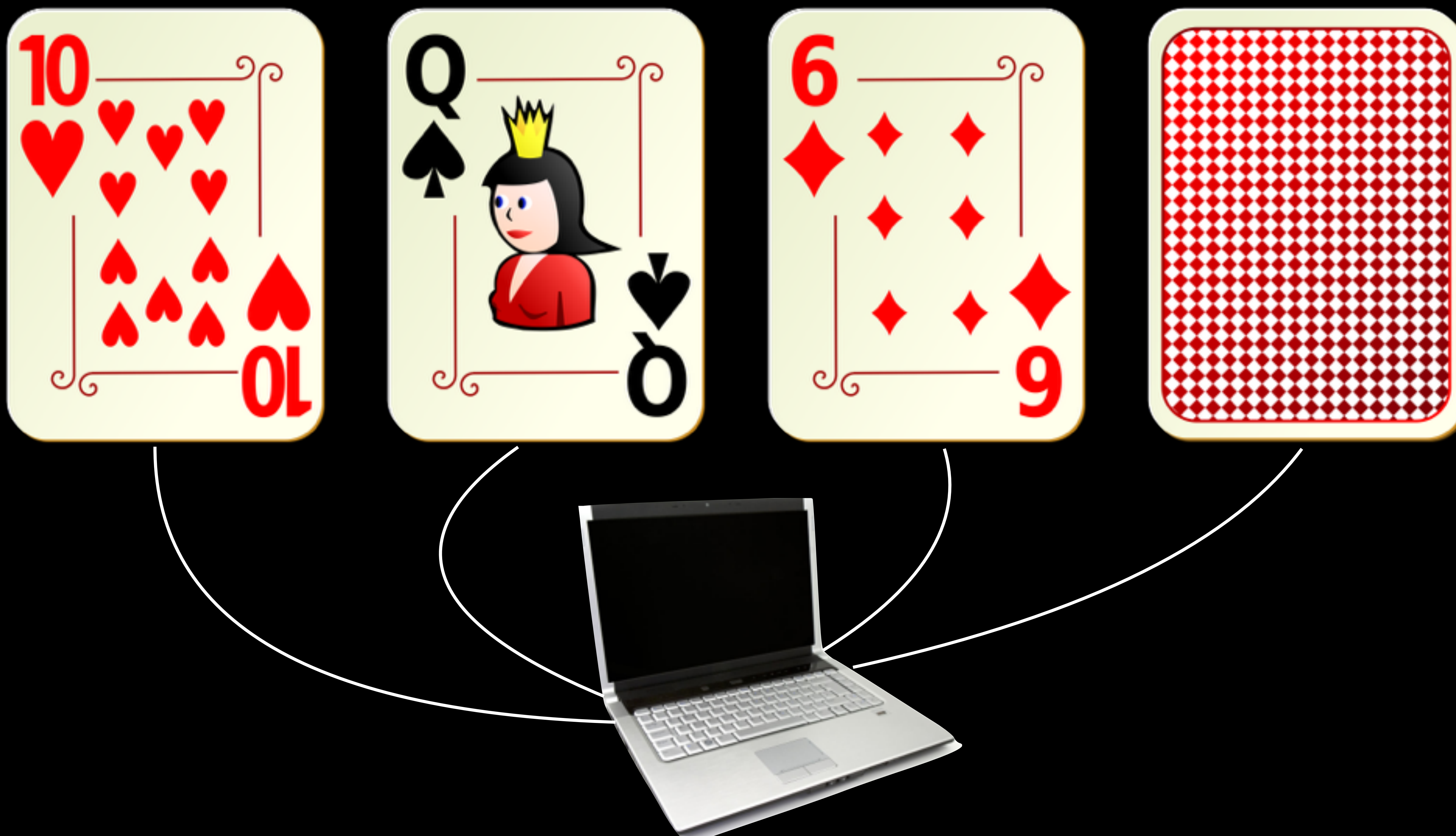

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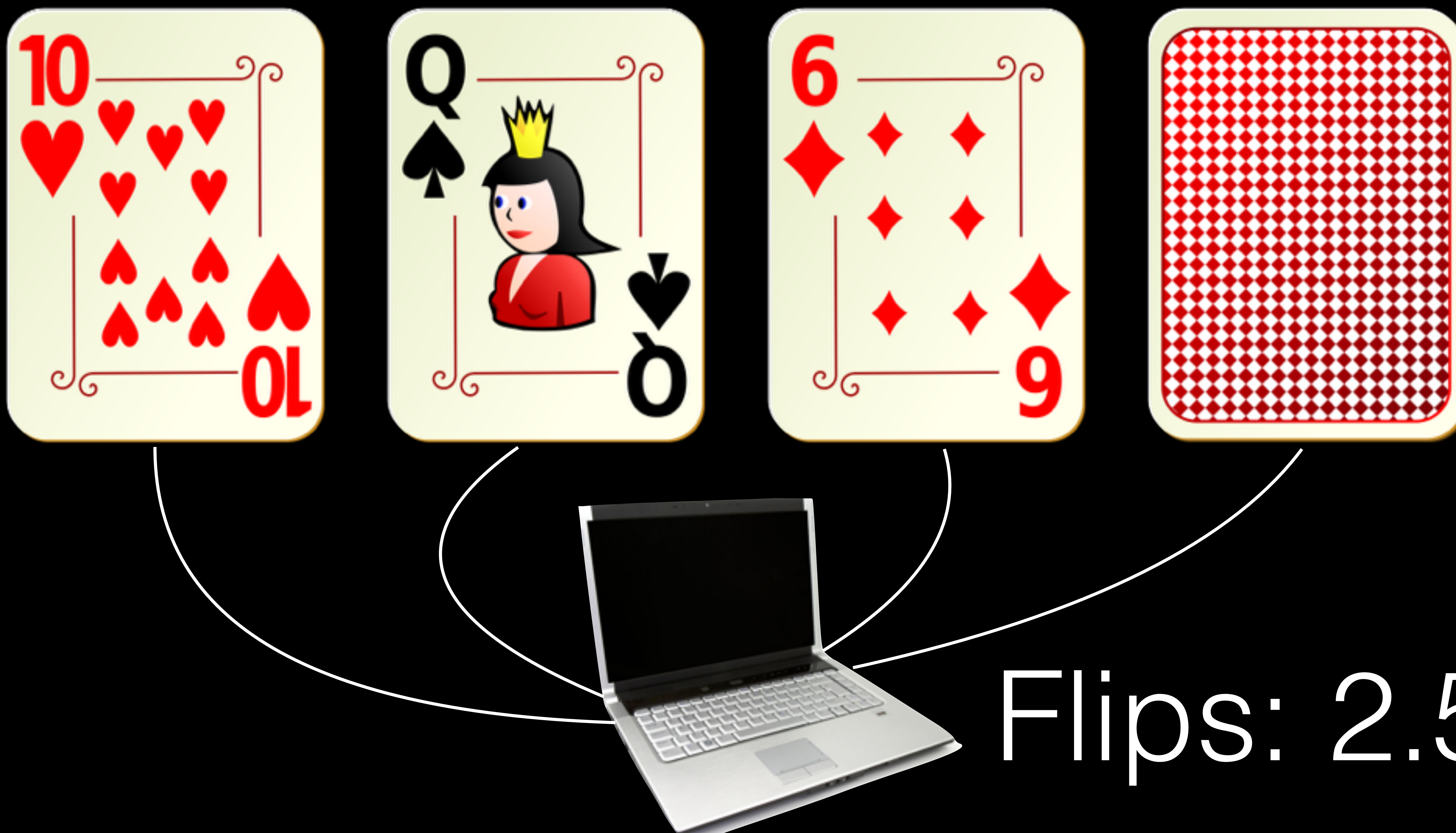

Flips: 2.5

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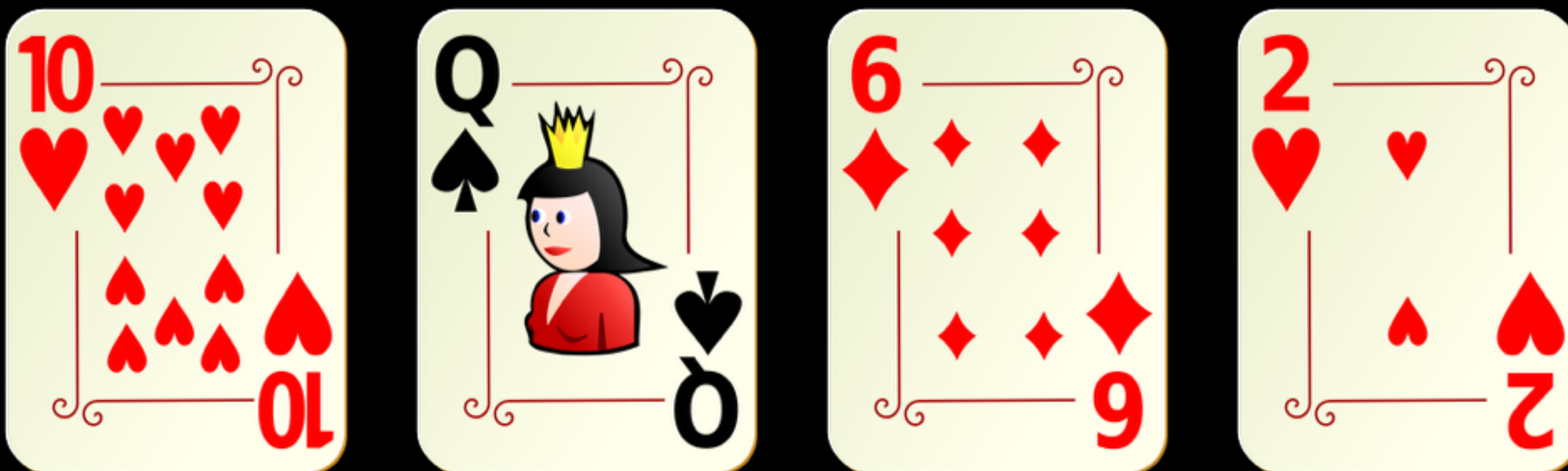

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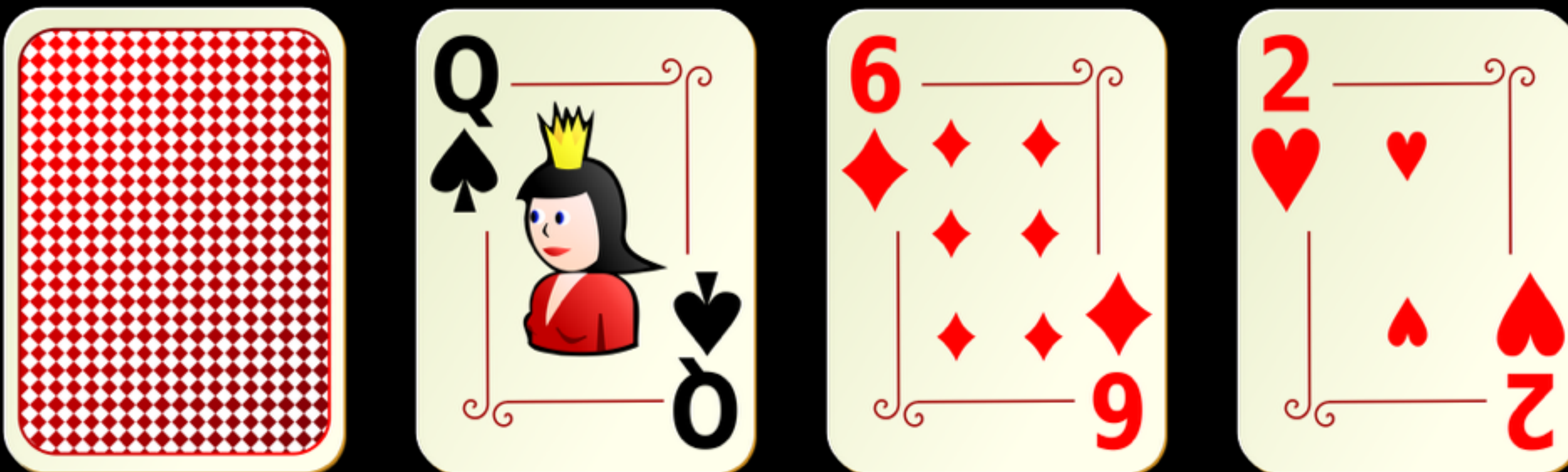

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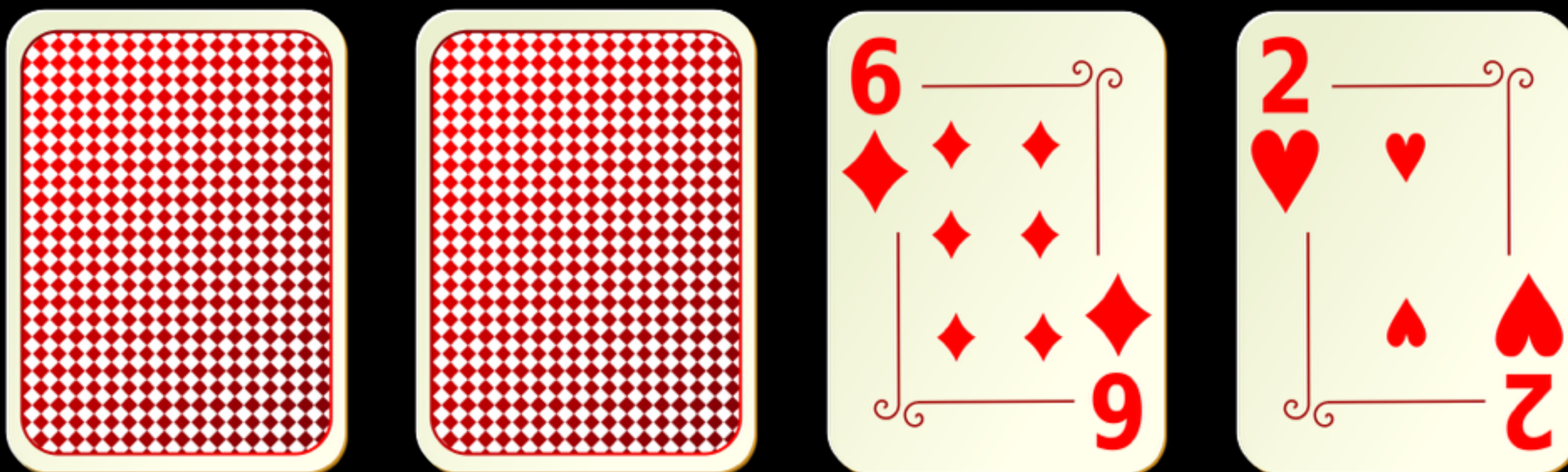

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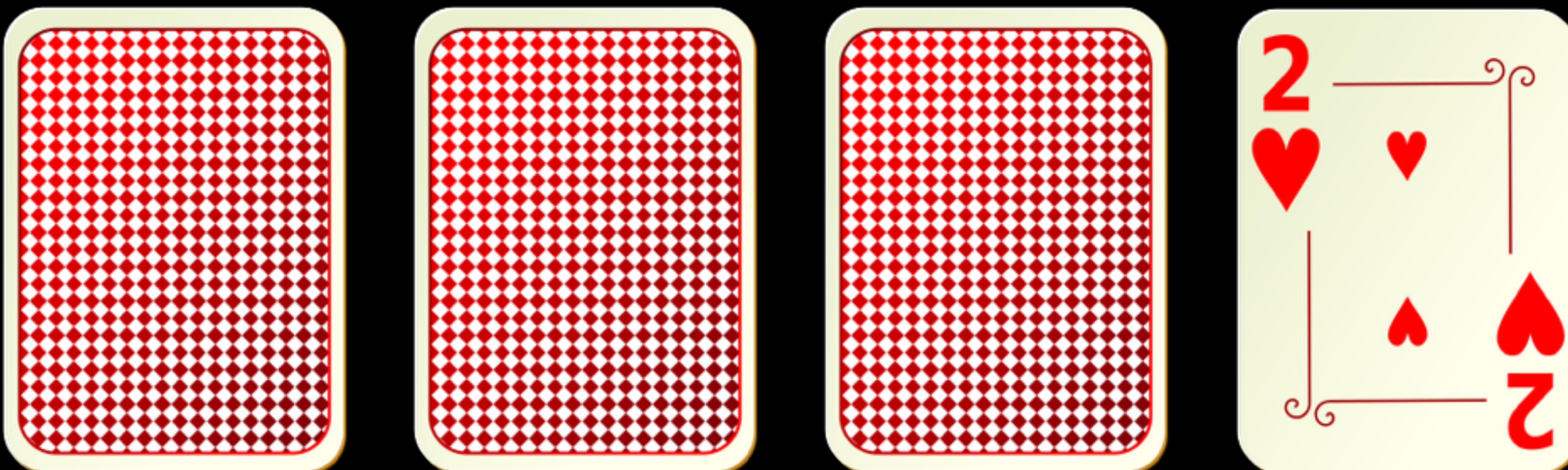

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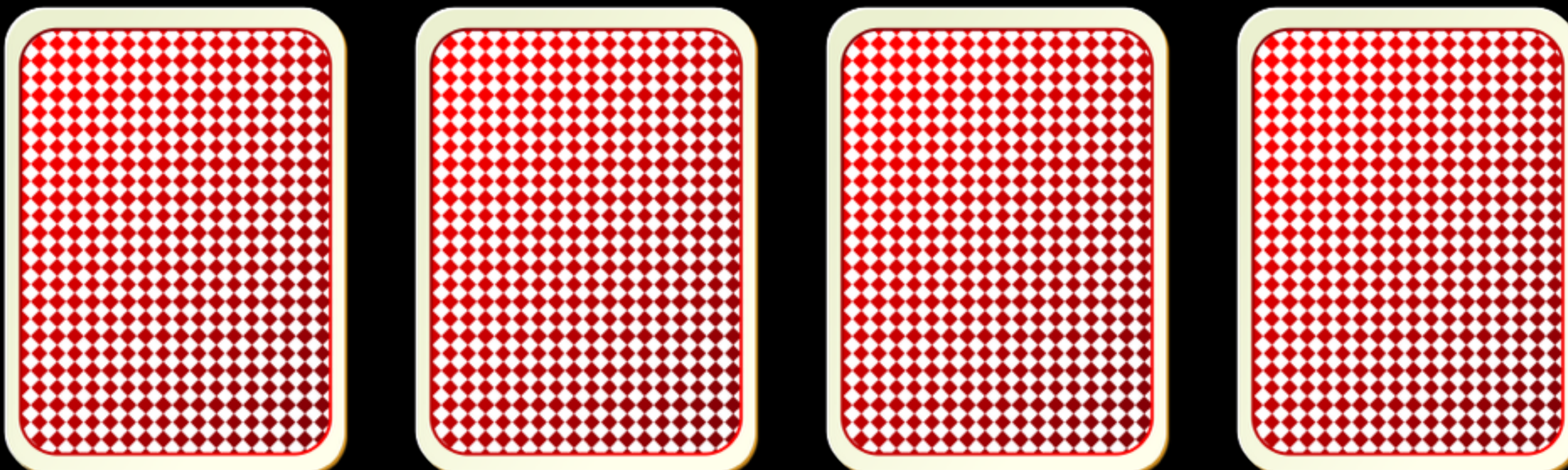

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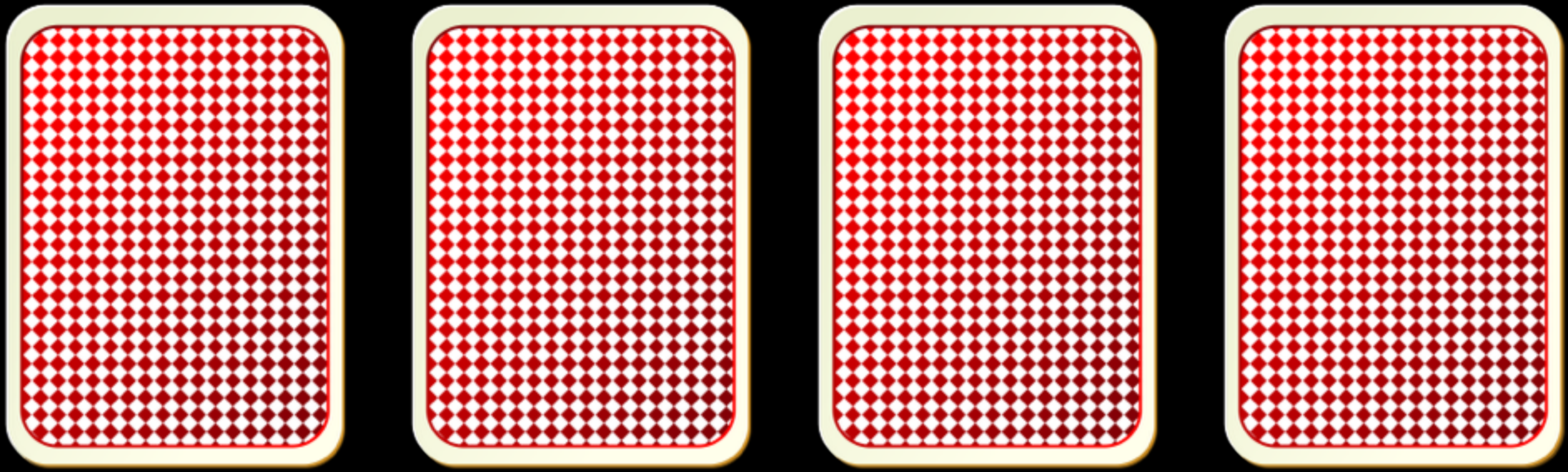

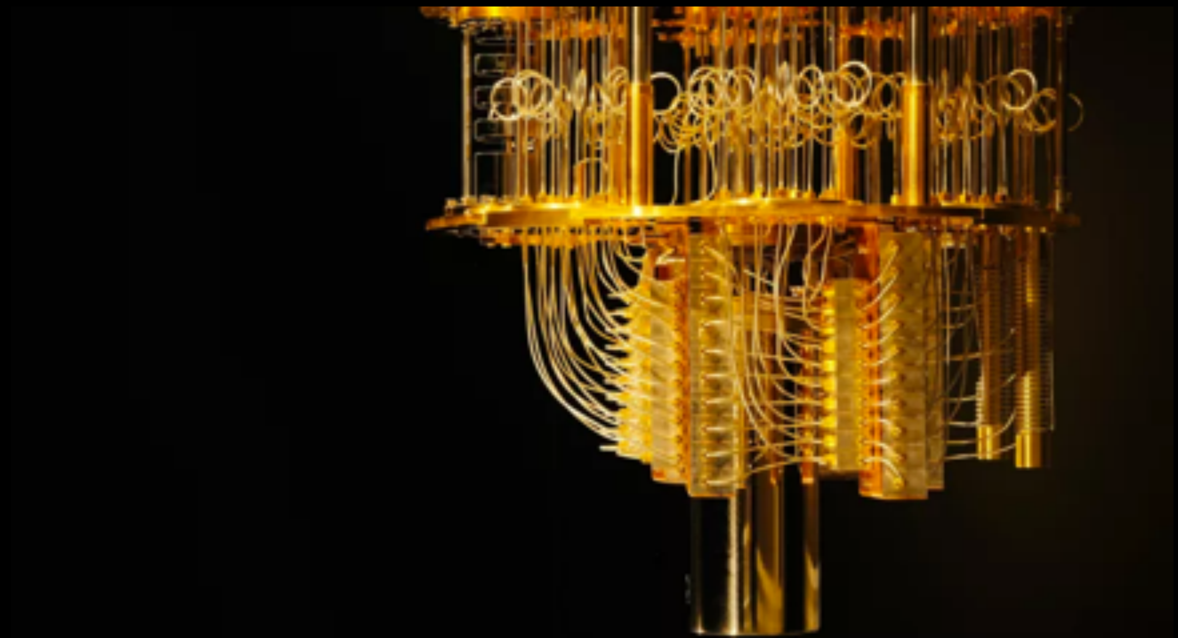

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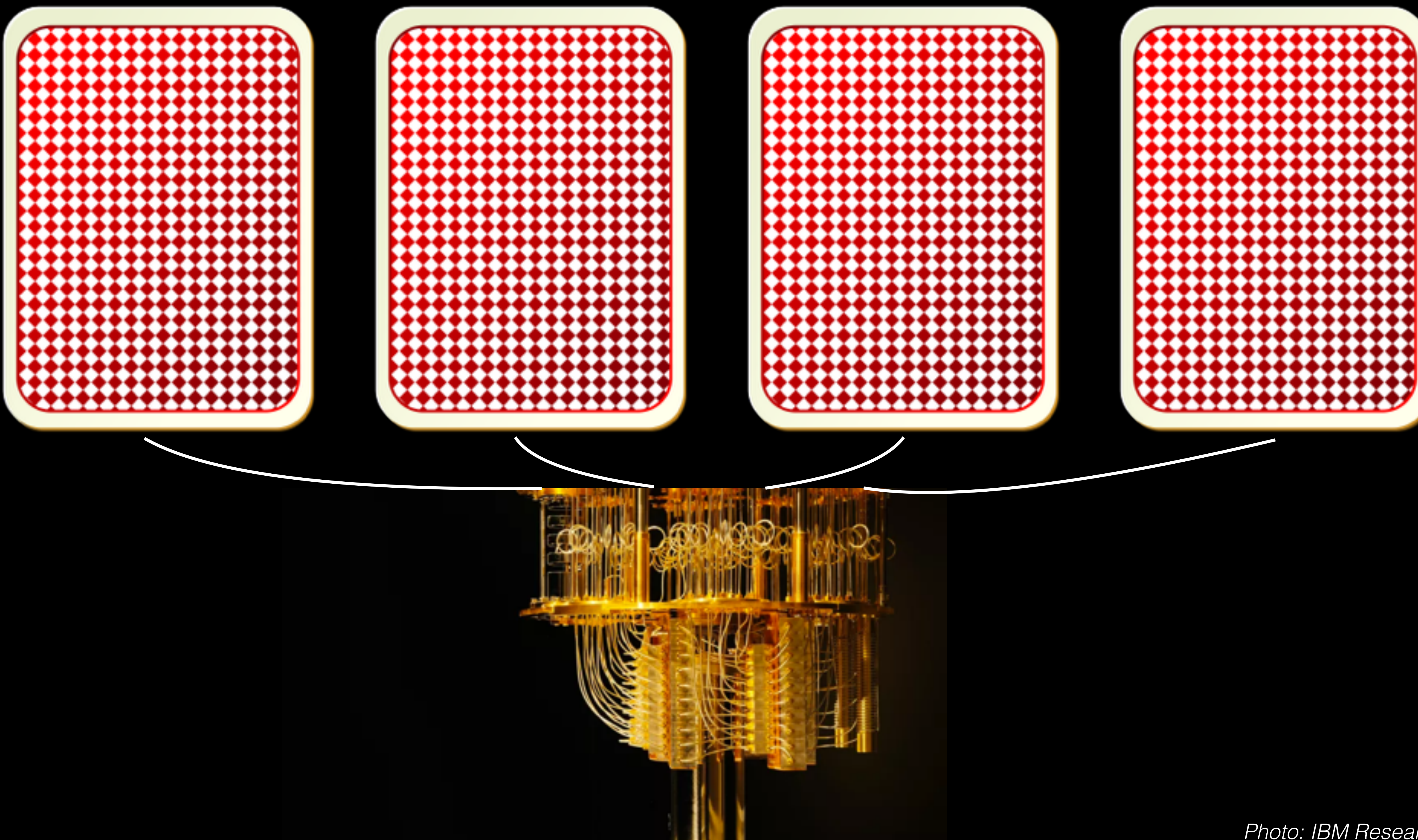

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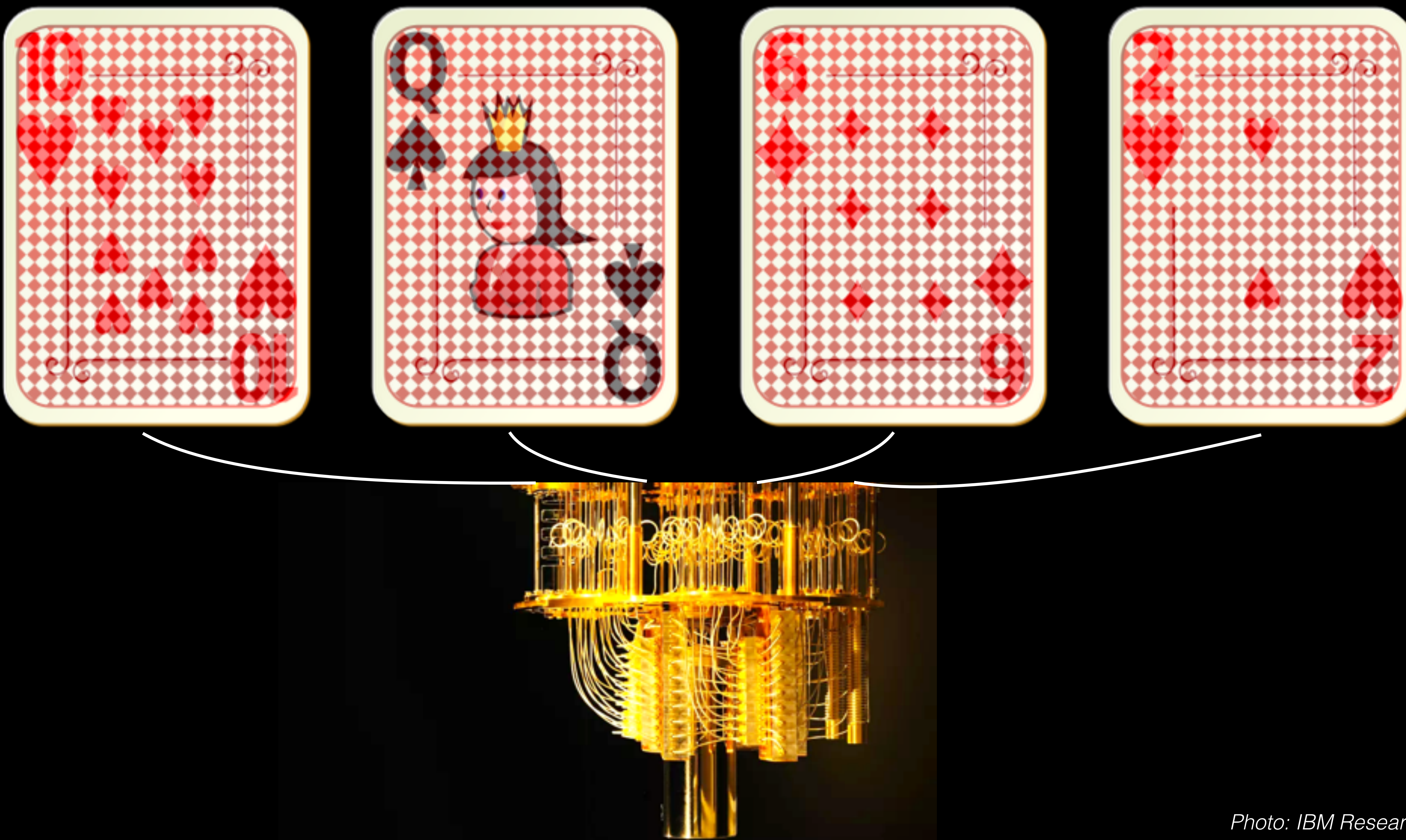

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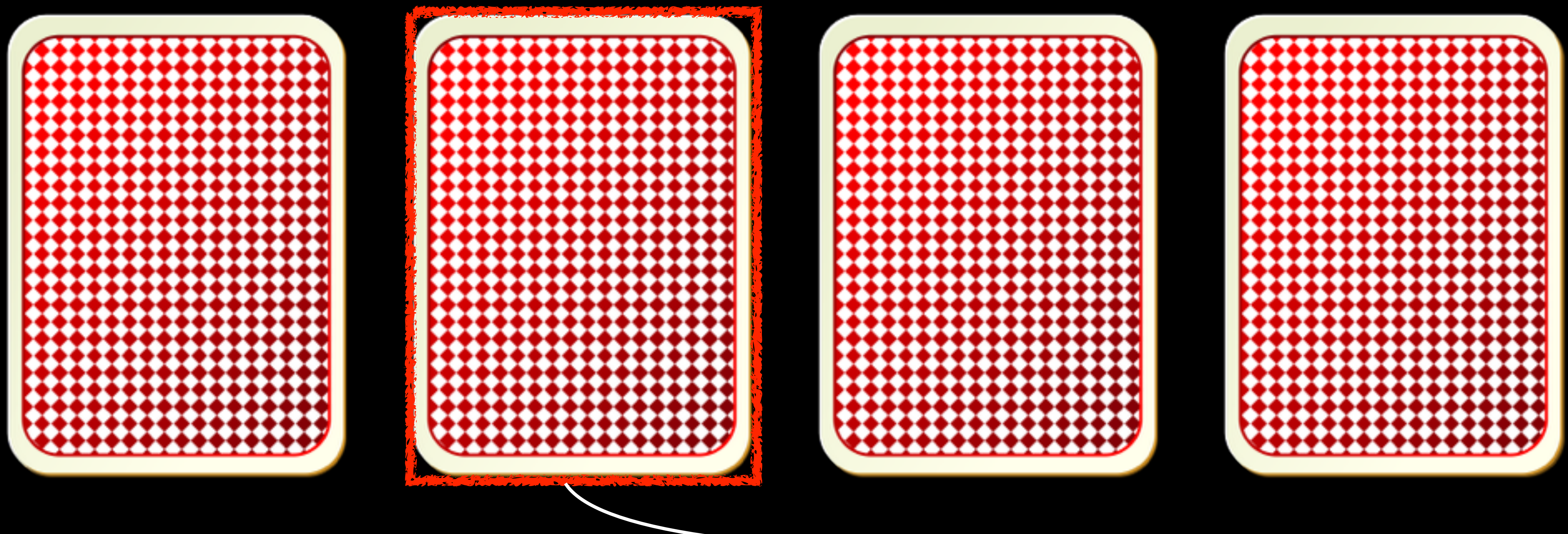

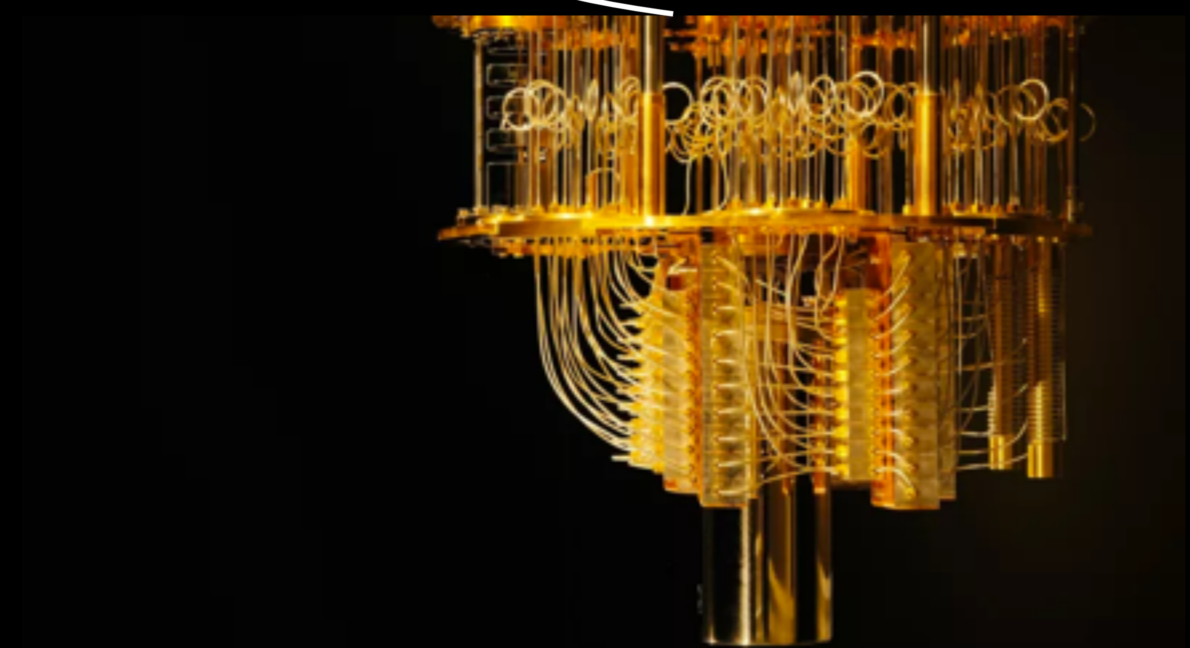

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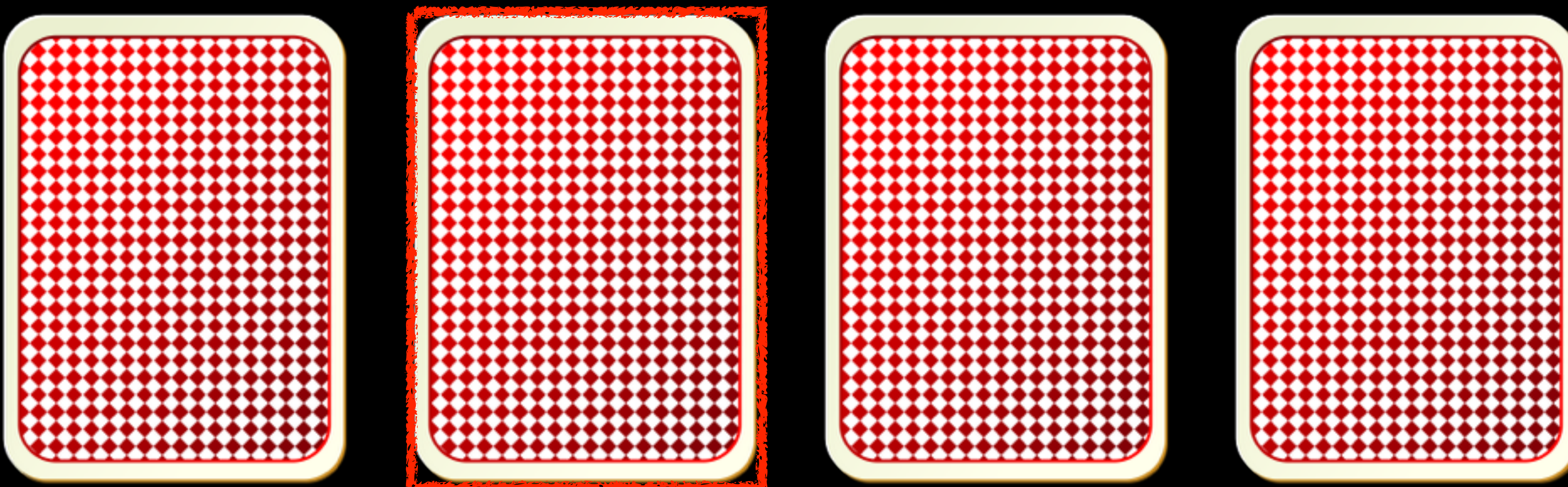

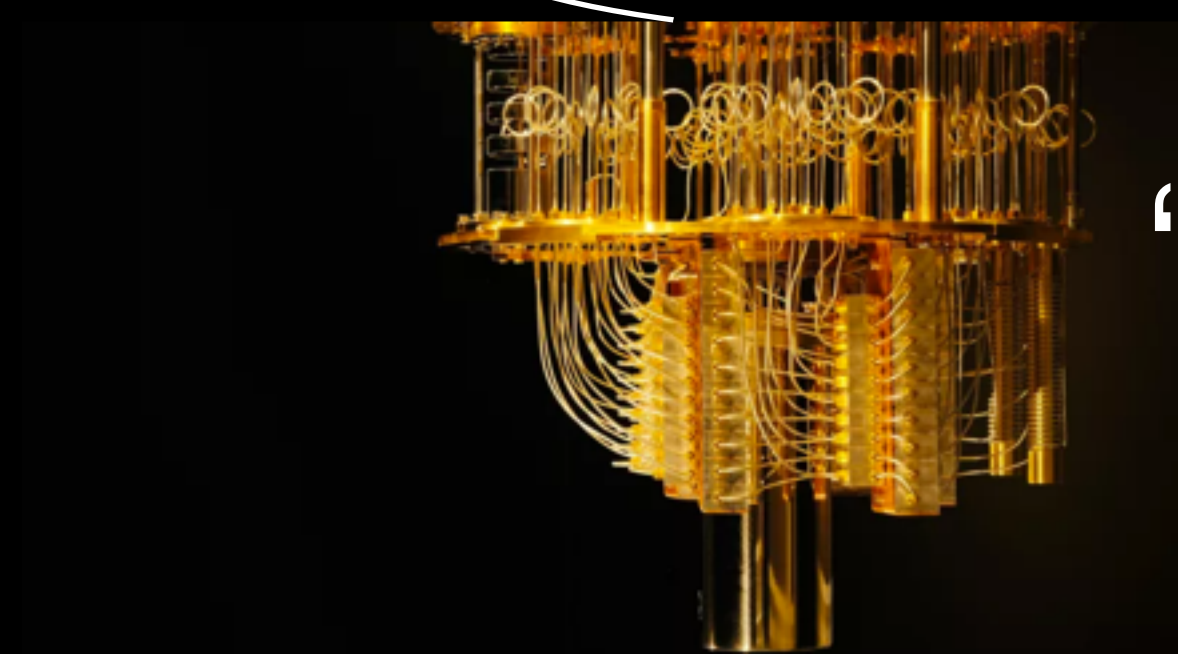

'Flips': 1

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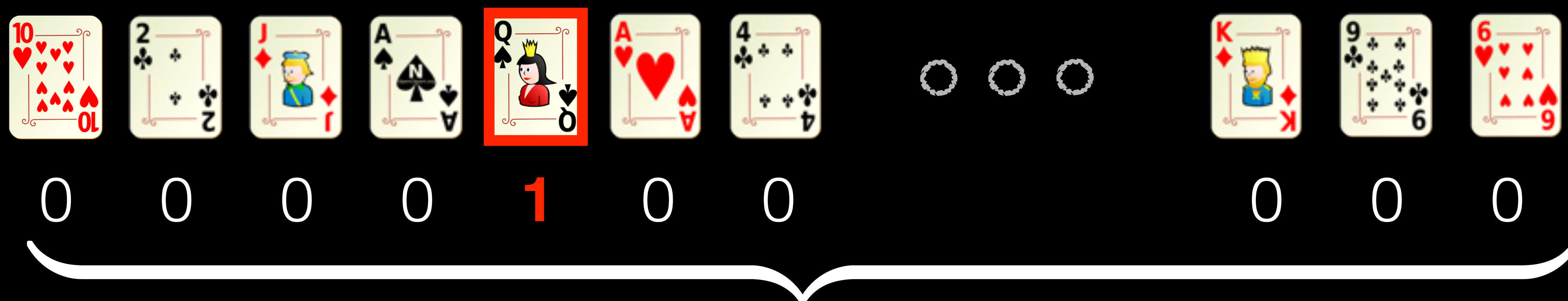
Solves the 'Unstructured Search' problem

Works in the black box / query setting

Find **1** marked item out of **N** items



N items

- Classical Computer: N queries

- Quantum Computer: $\sim\sqrt{N}$ queries

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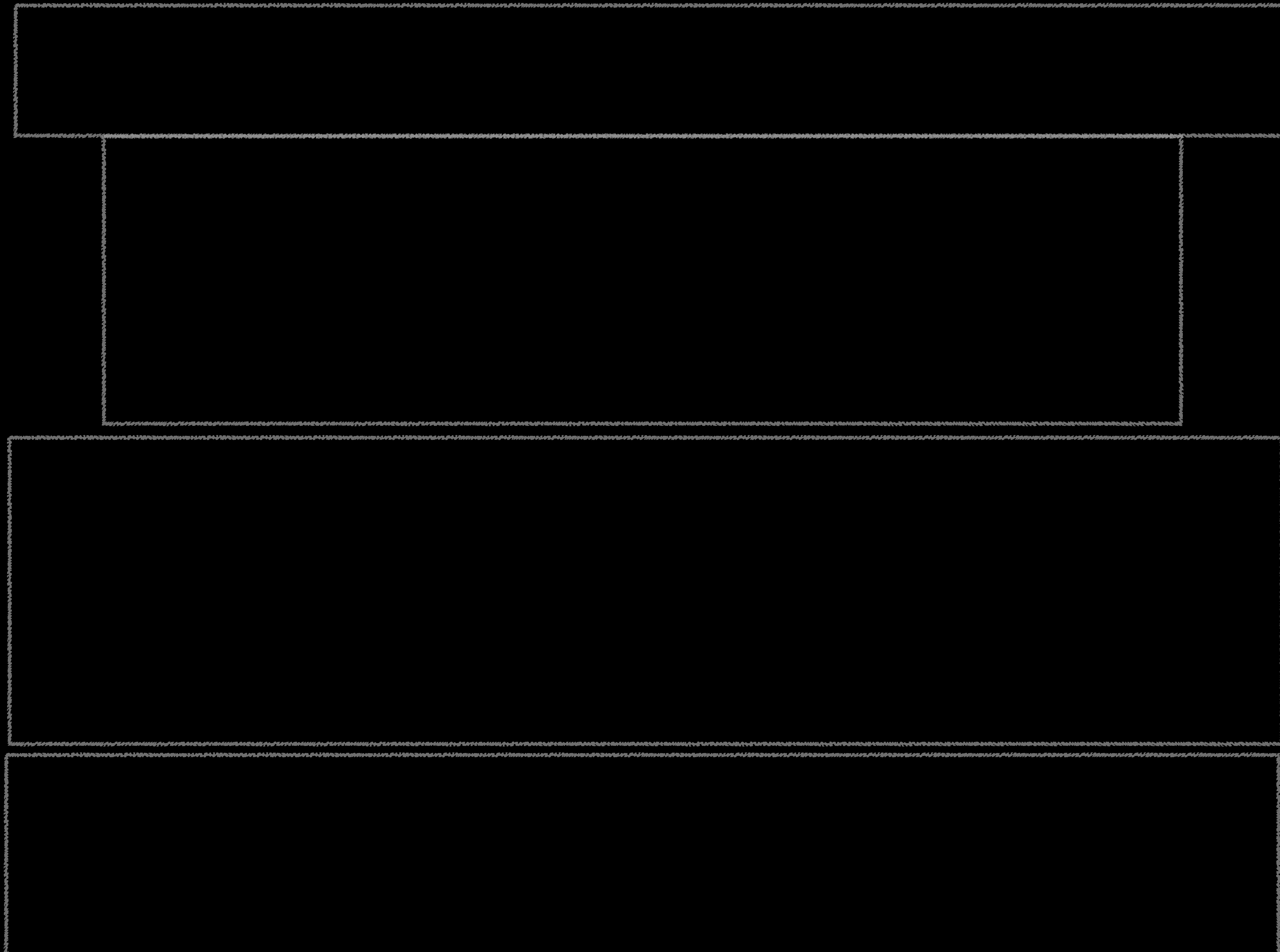
$$\langle\psi|\phi\rangle = \cos\theta$$



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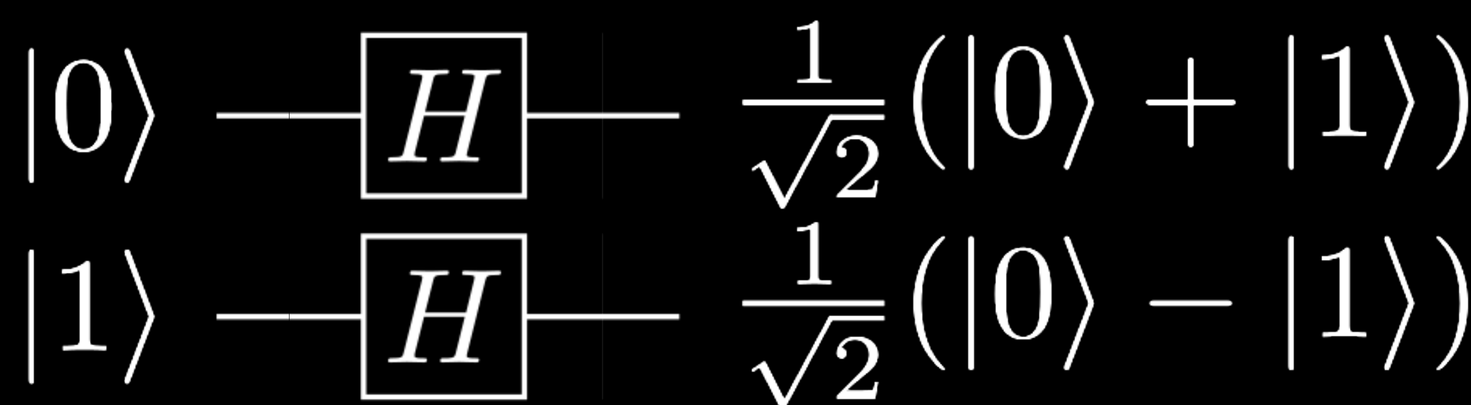

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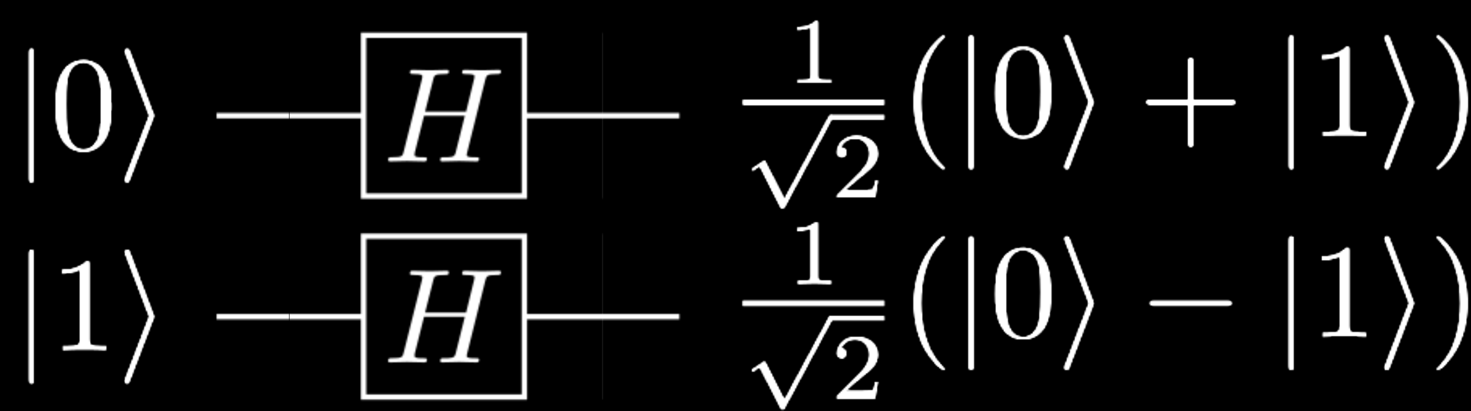

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Hadamard gates on all n qubits

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Using U_0 we define the operator

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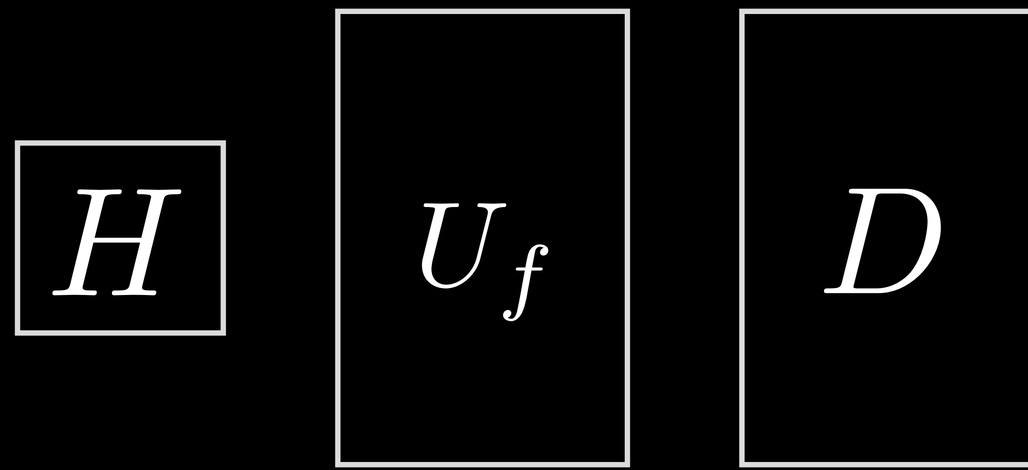
H


H
U_f


H
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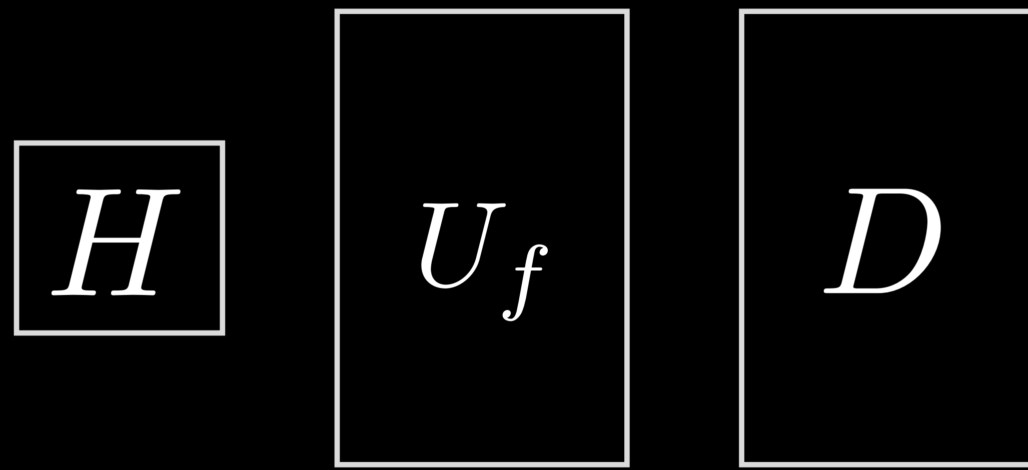
D



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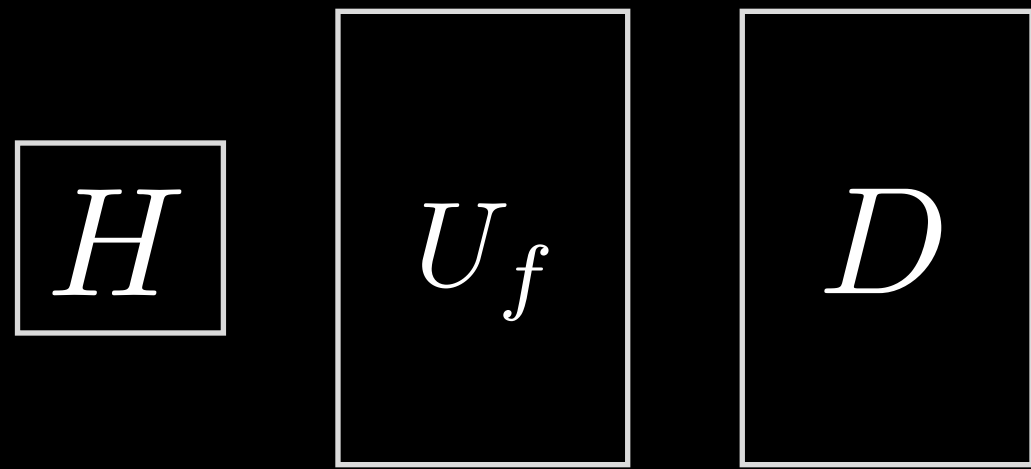

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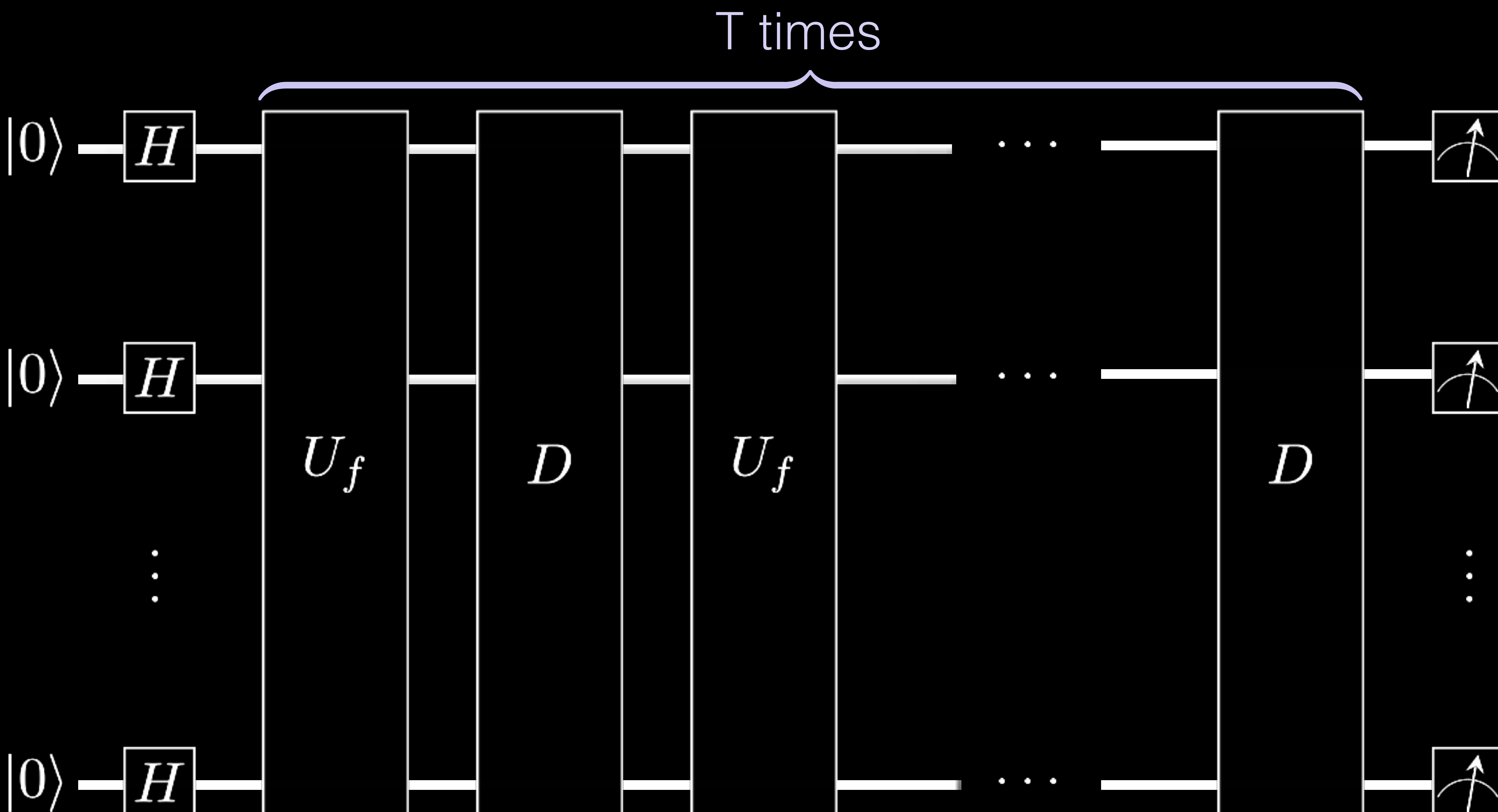

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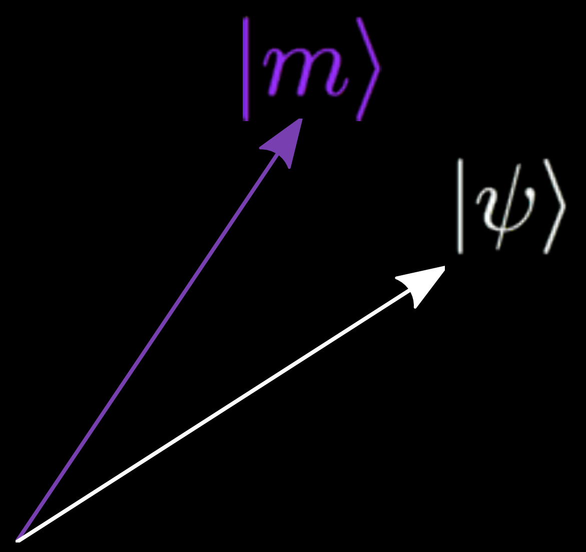

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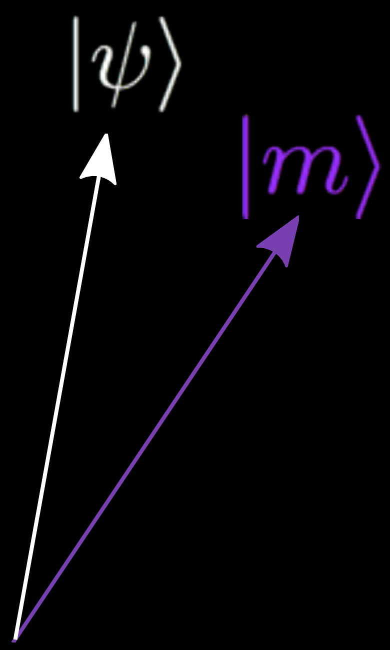

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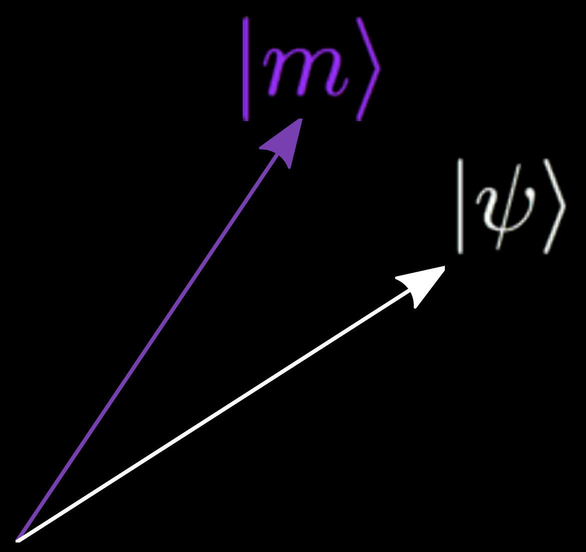

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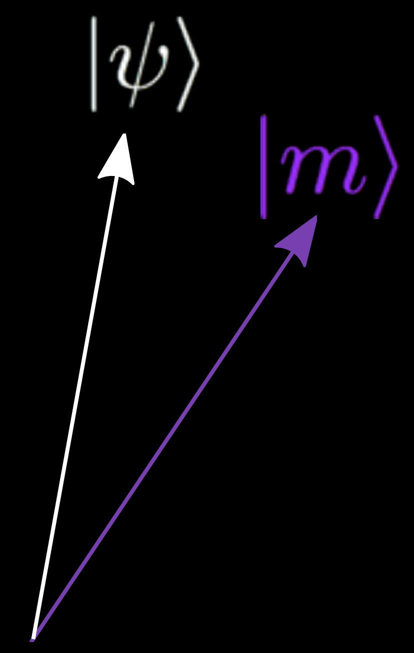

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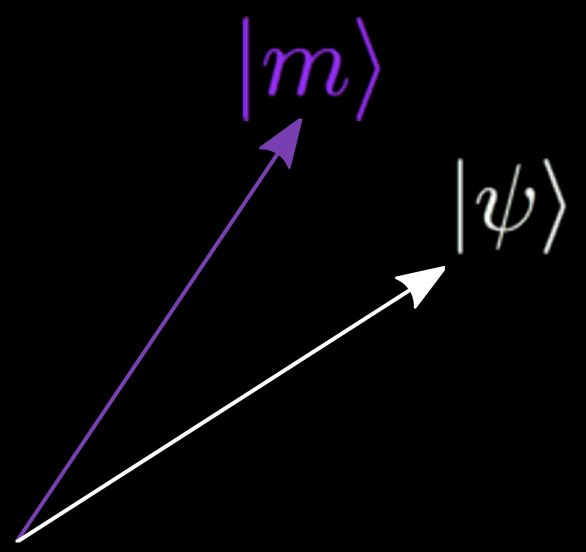

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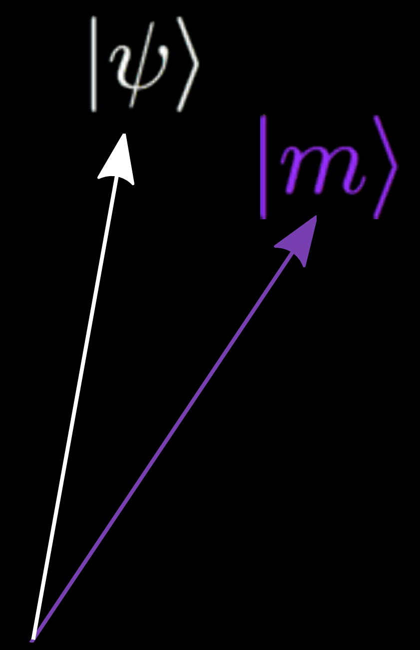

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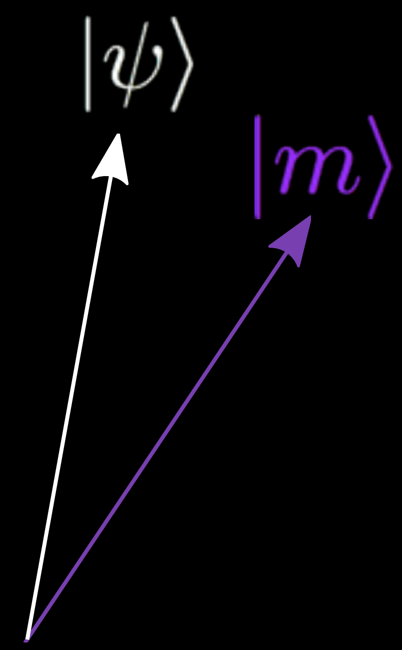

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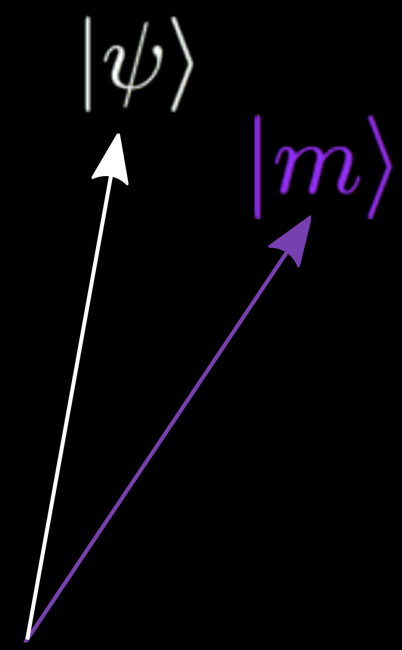

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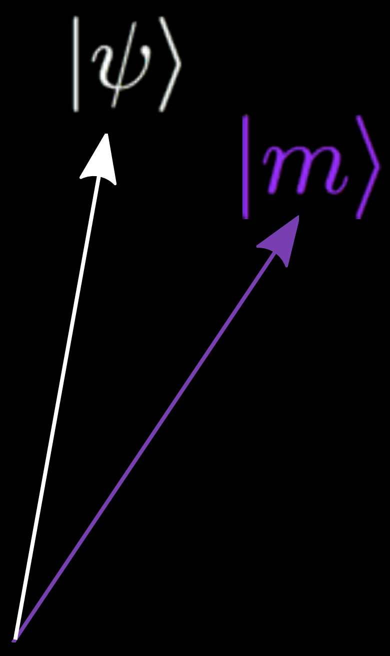

$$U_f|x\rangle \mapsto \begin{cases} |x\rangle & \text{if } x \text{ is the marked item} \\ -|x\rangle & \text{Otherwise} \end{cases}$$

Reflection about the marked item $|m\rangle$

$$H^{\otimes n}|0\rangle = \frac{1}{\sqrt{2^n}} \sum_x |x\rangle = |+\rangle$$

$$D = \quad H^{\otimes n} U_0 H^{\otimes n}$$

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Why does this work?

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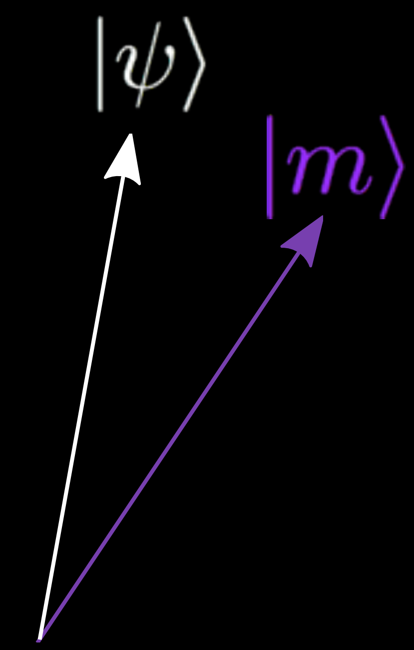

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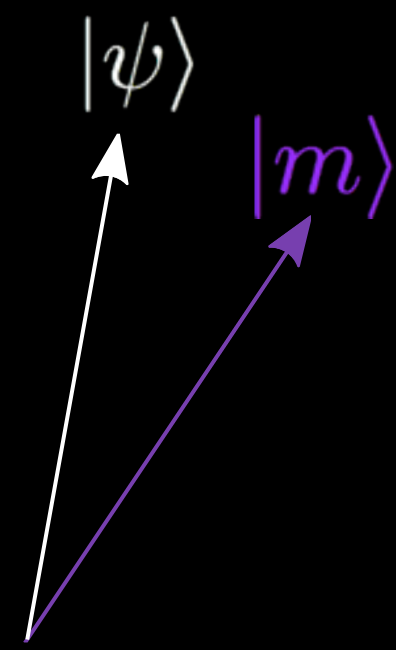

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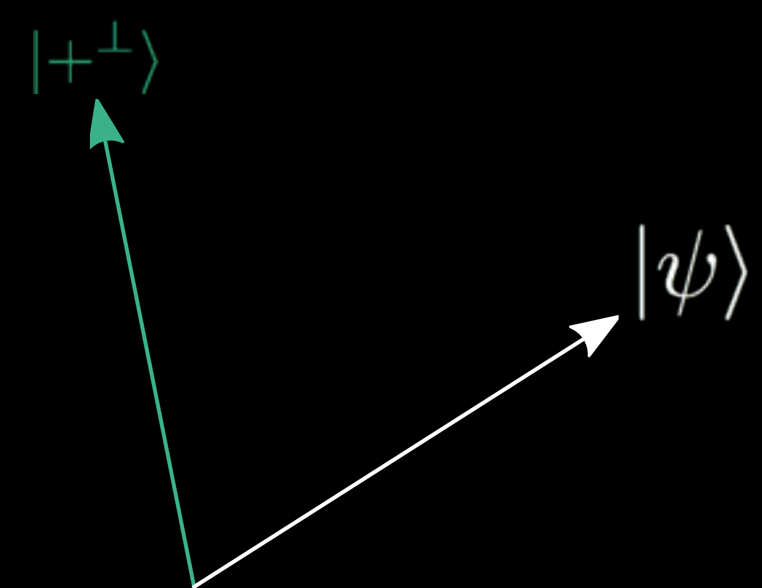

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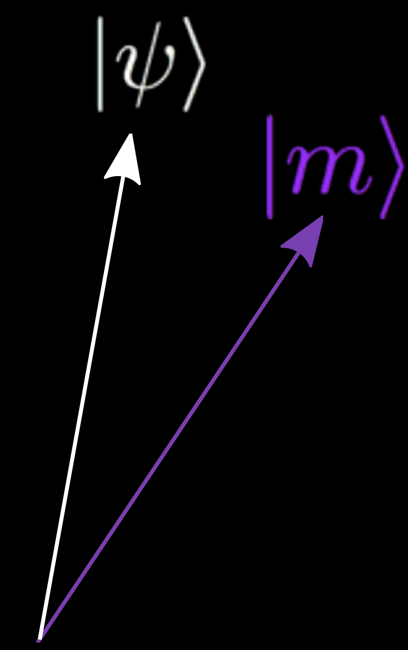

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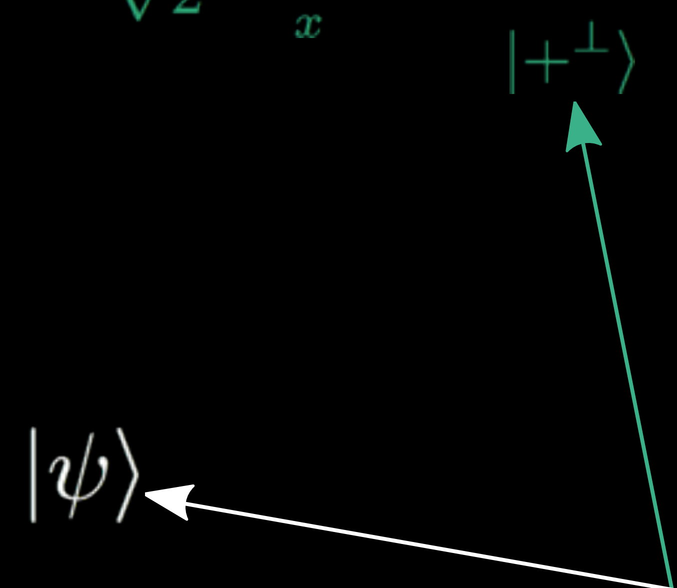

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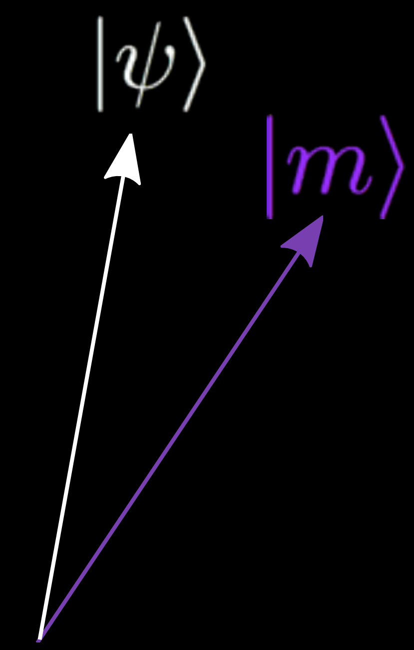

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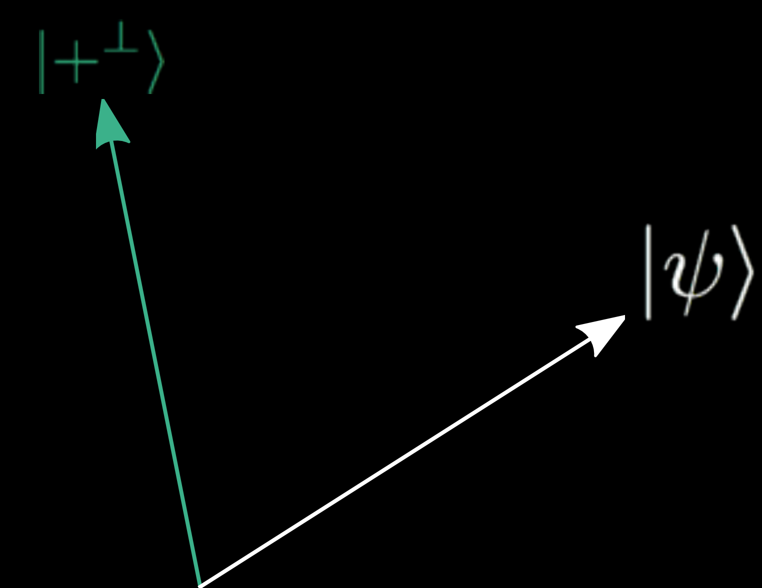

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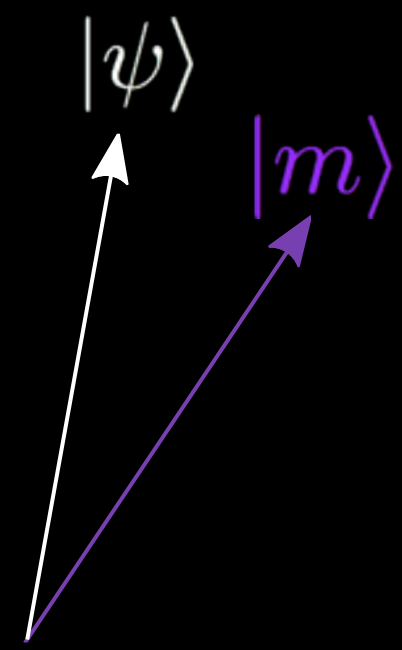

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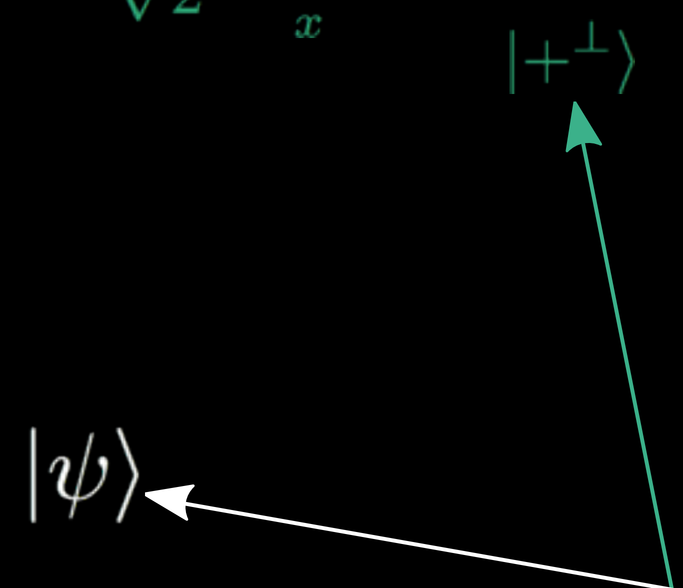

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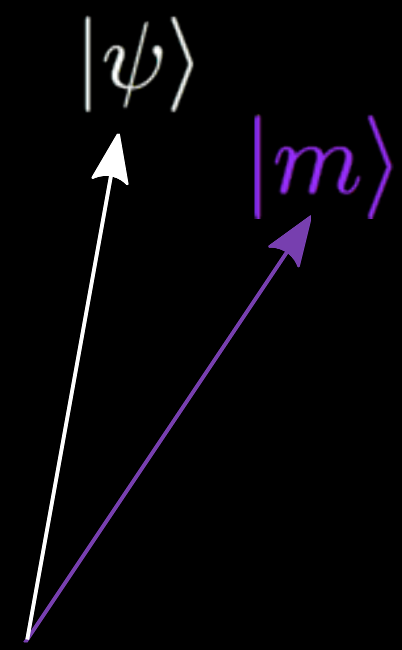

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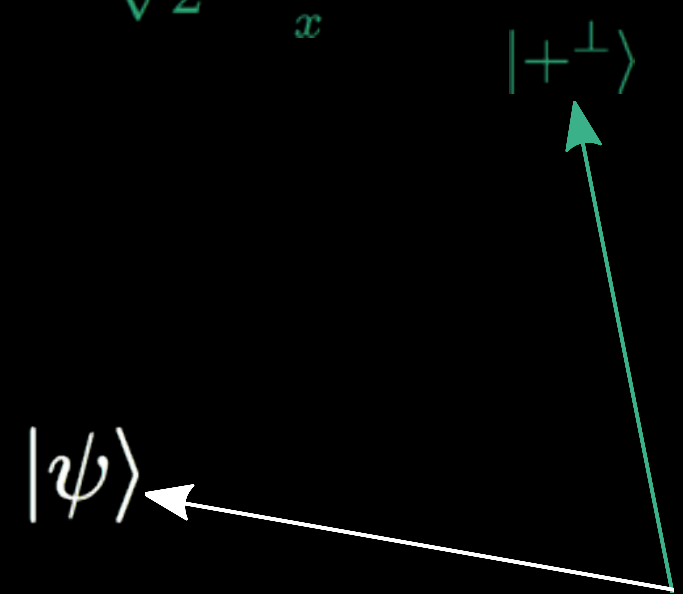

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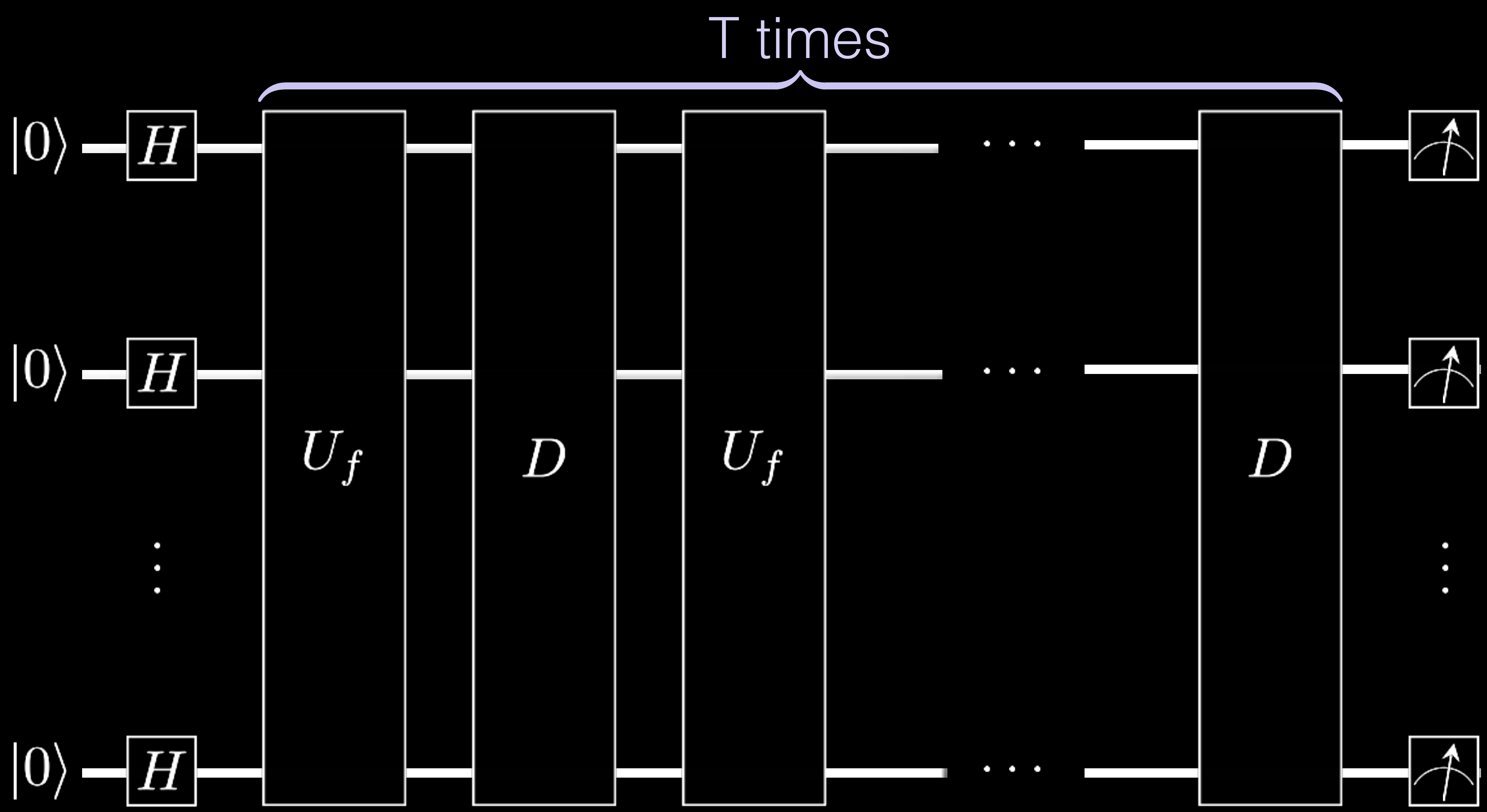

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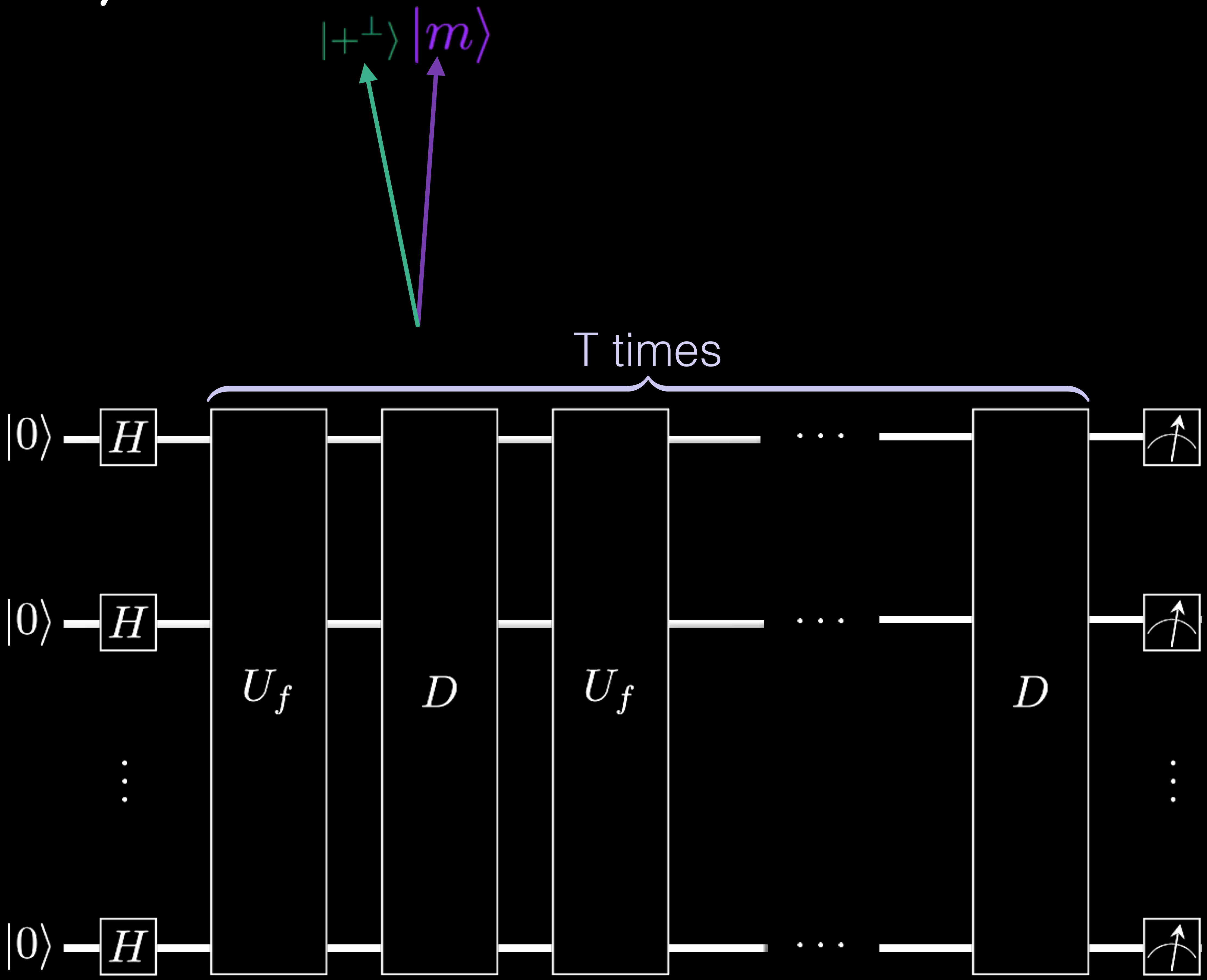

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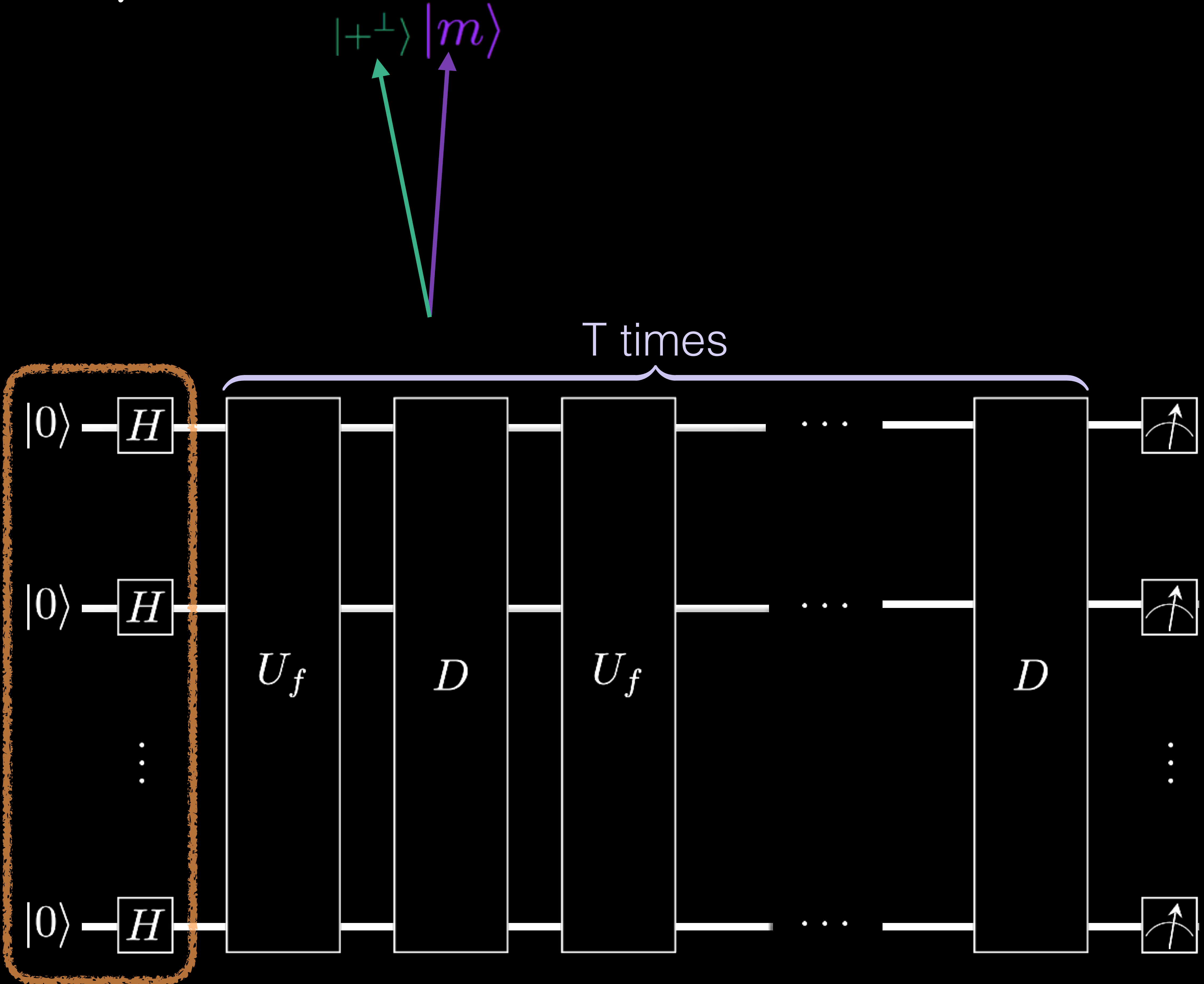

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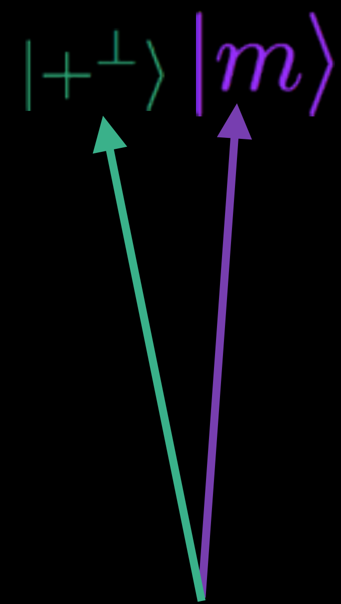

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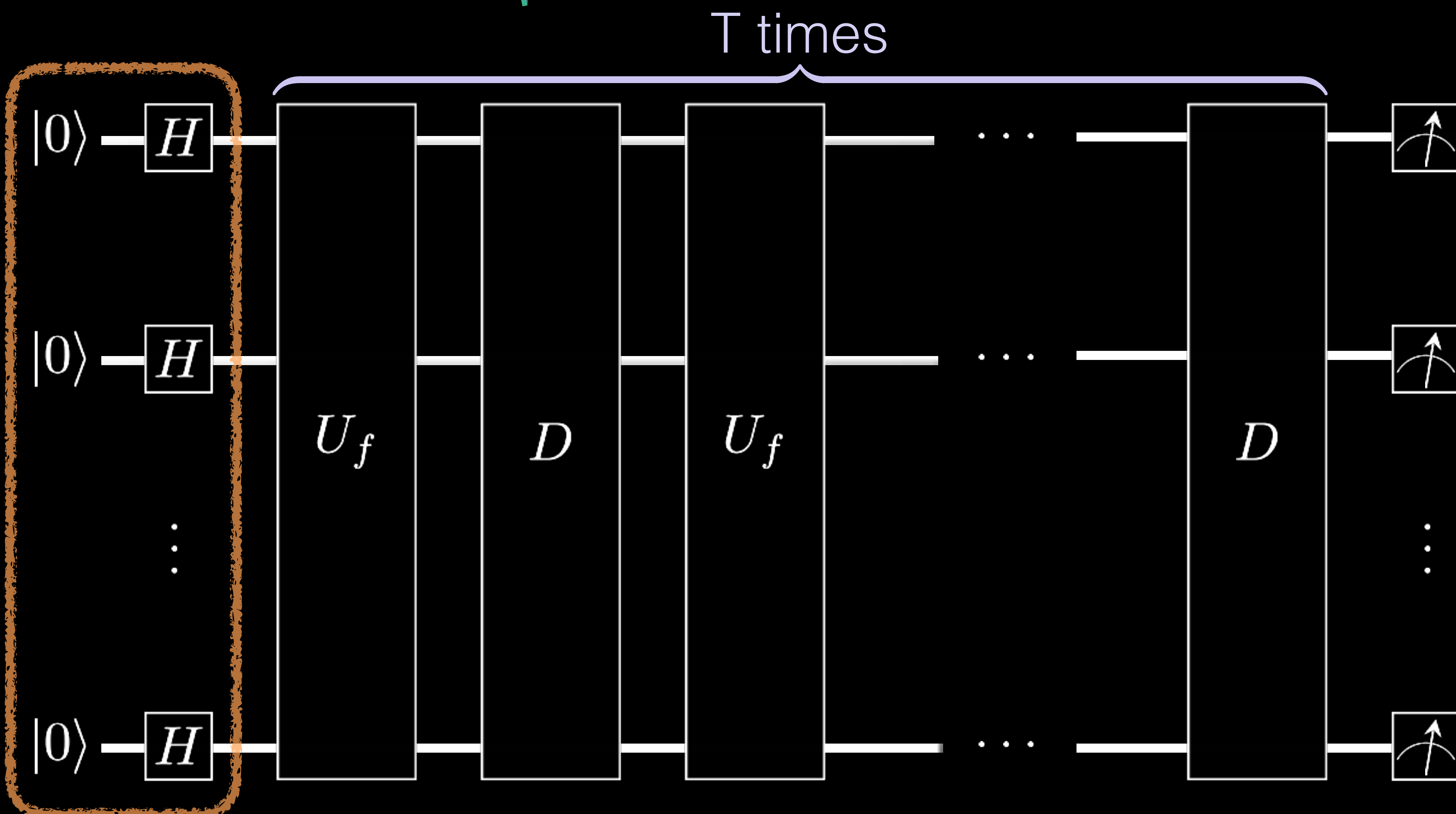

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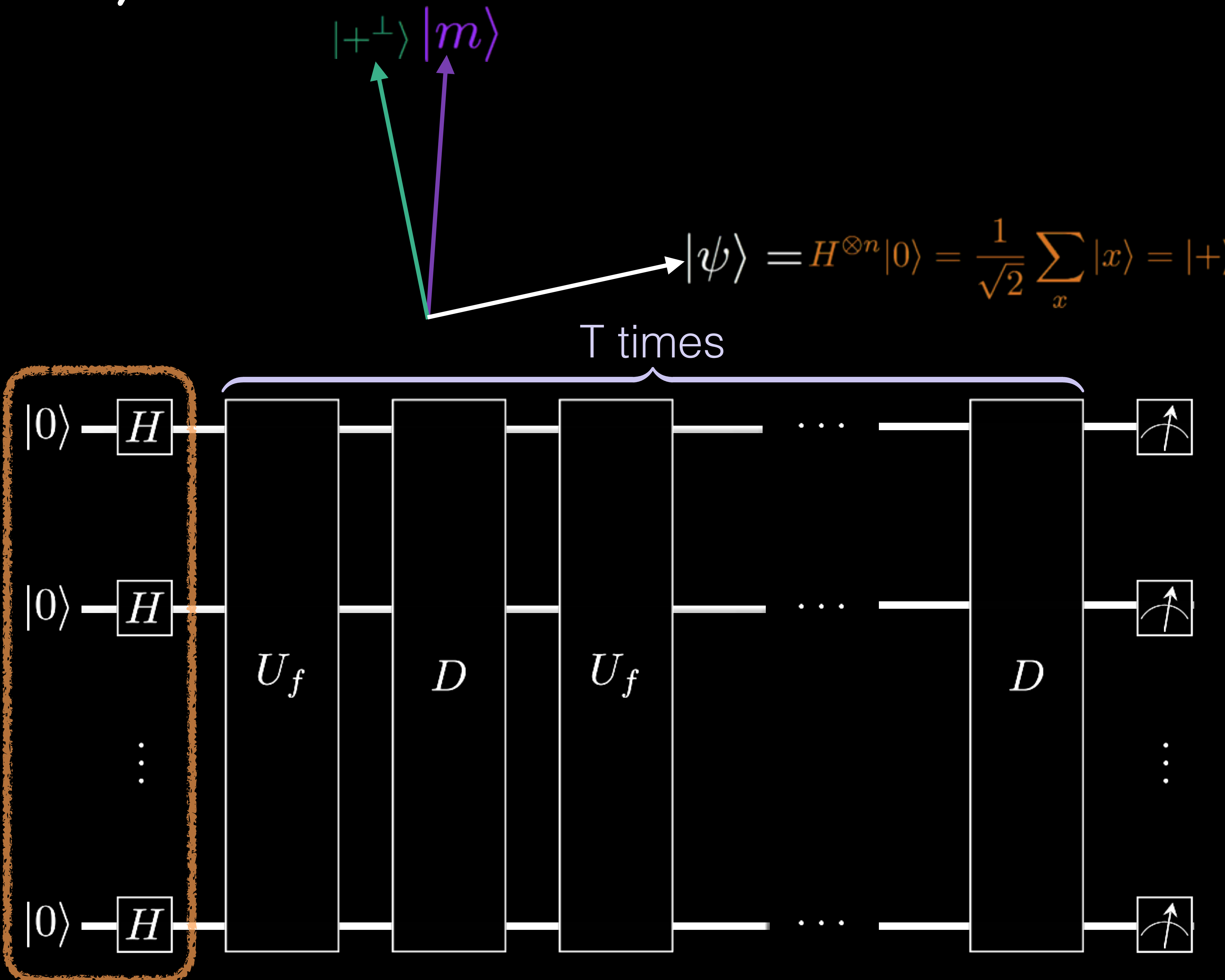

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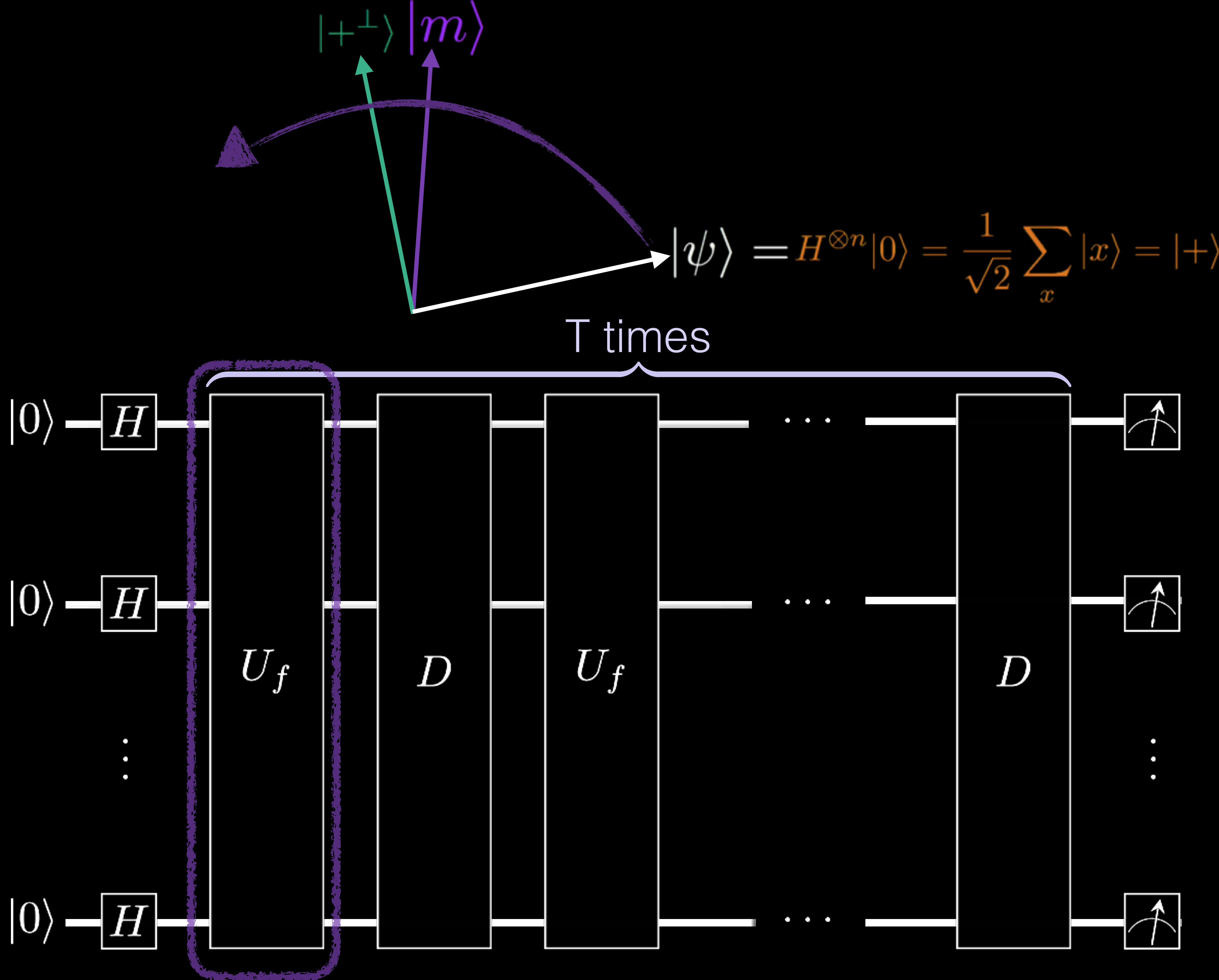

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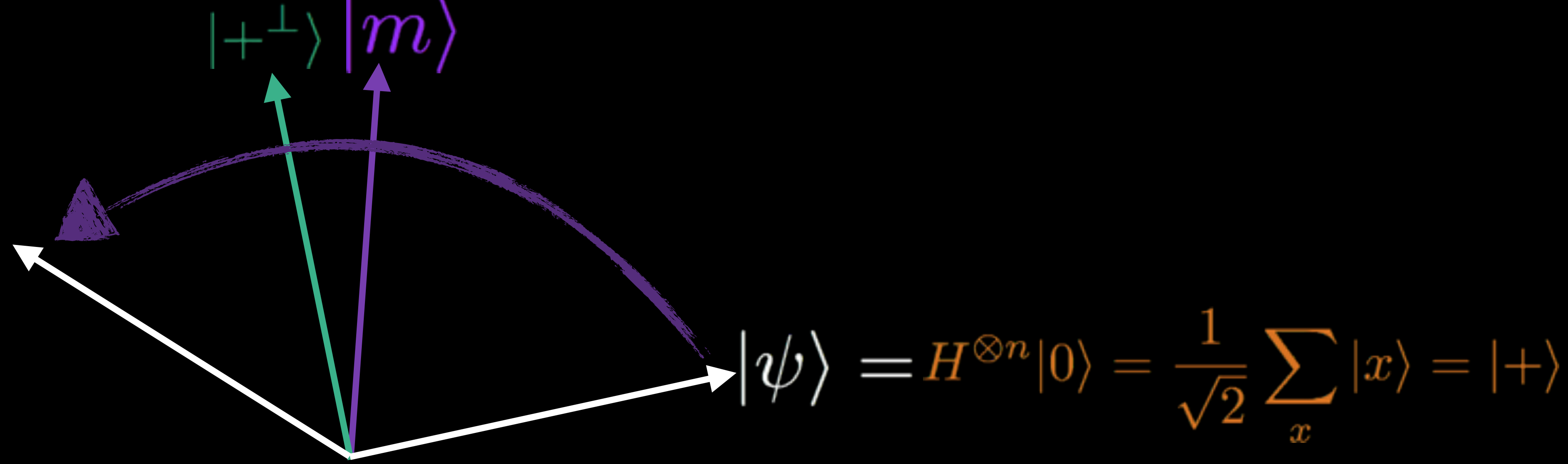

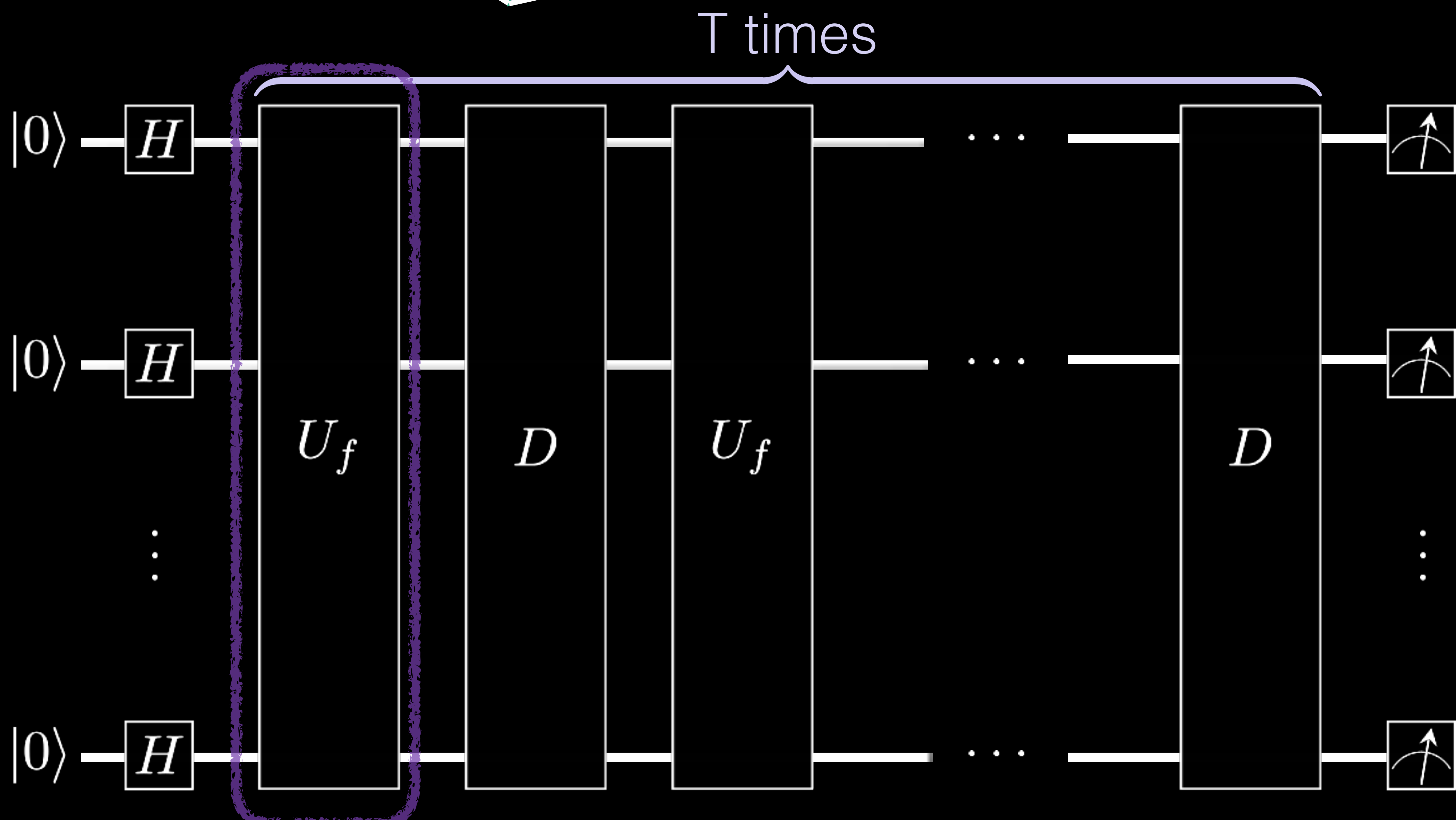

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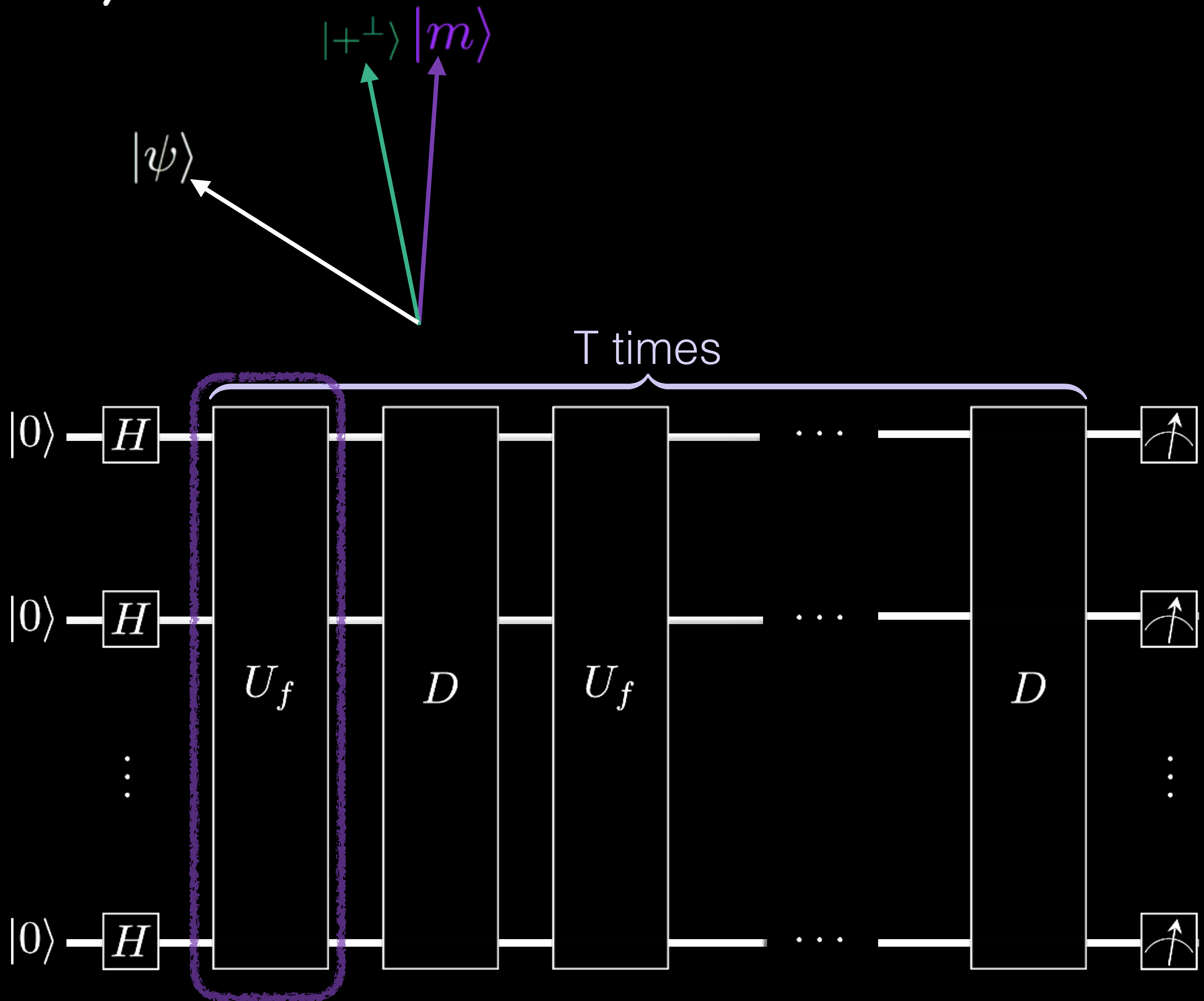

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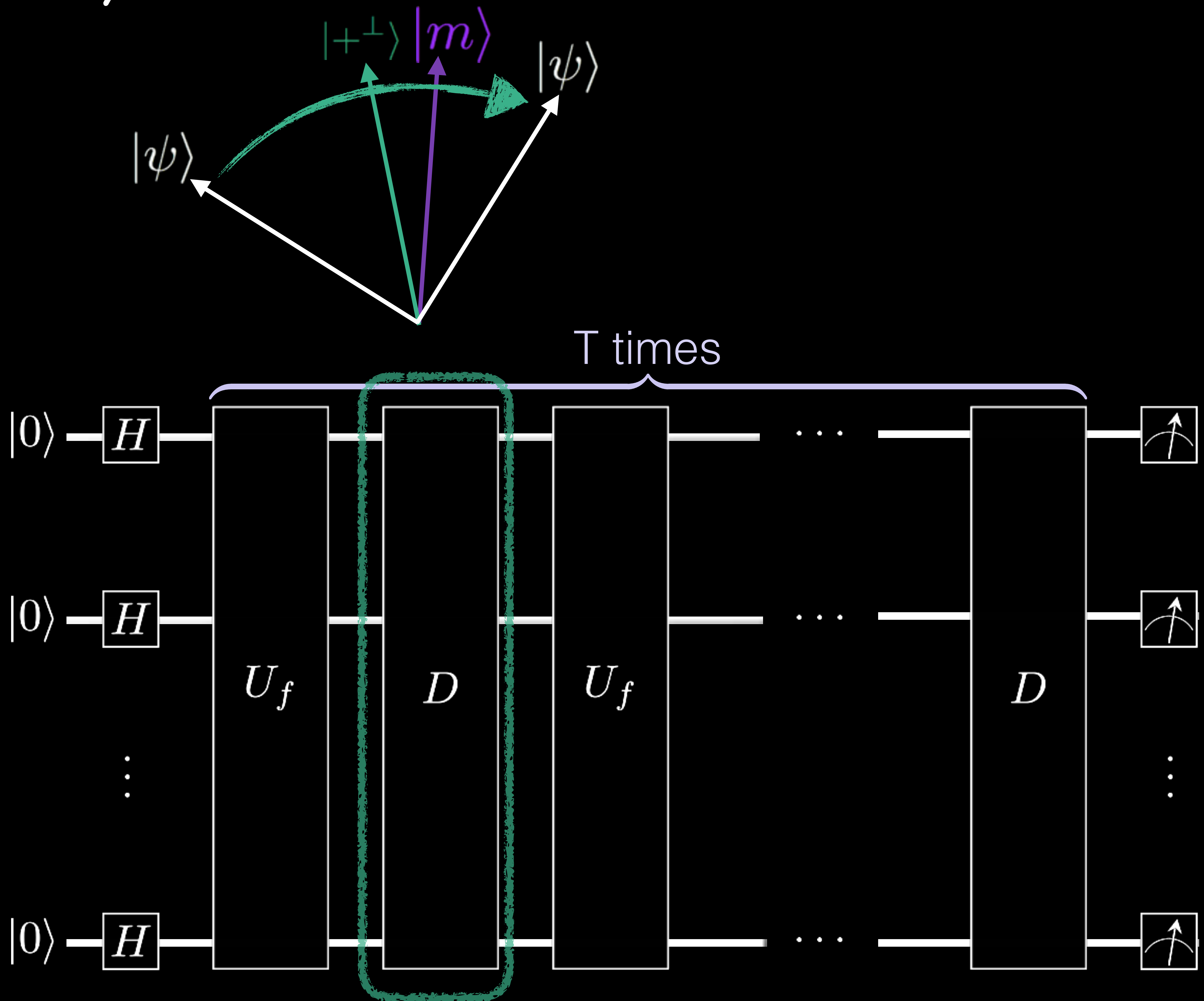

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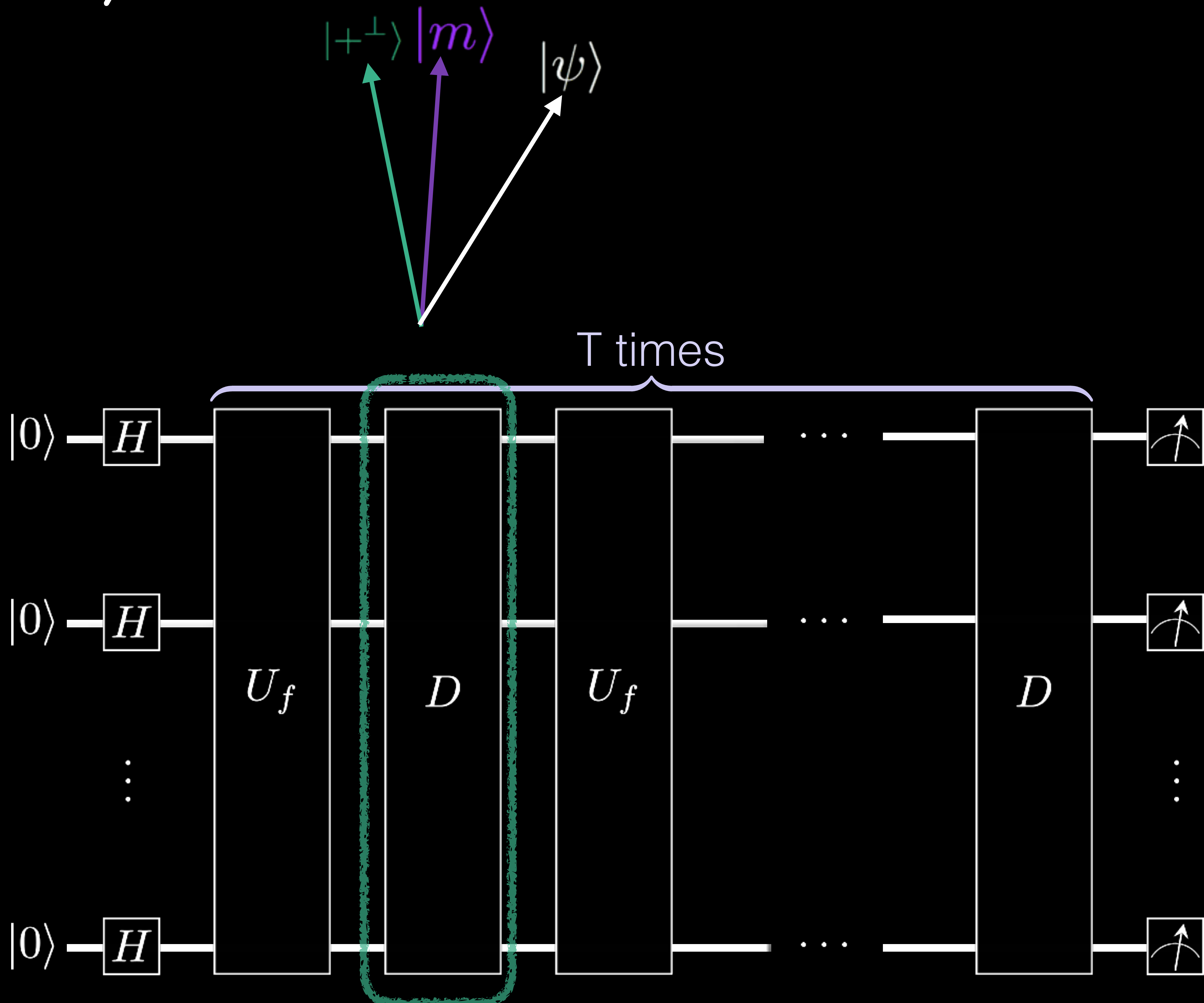

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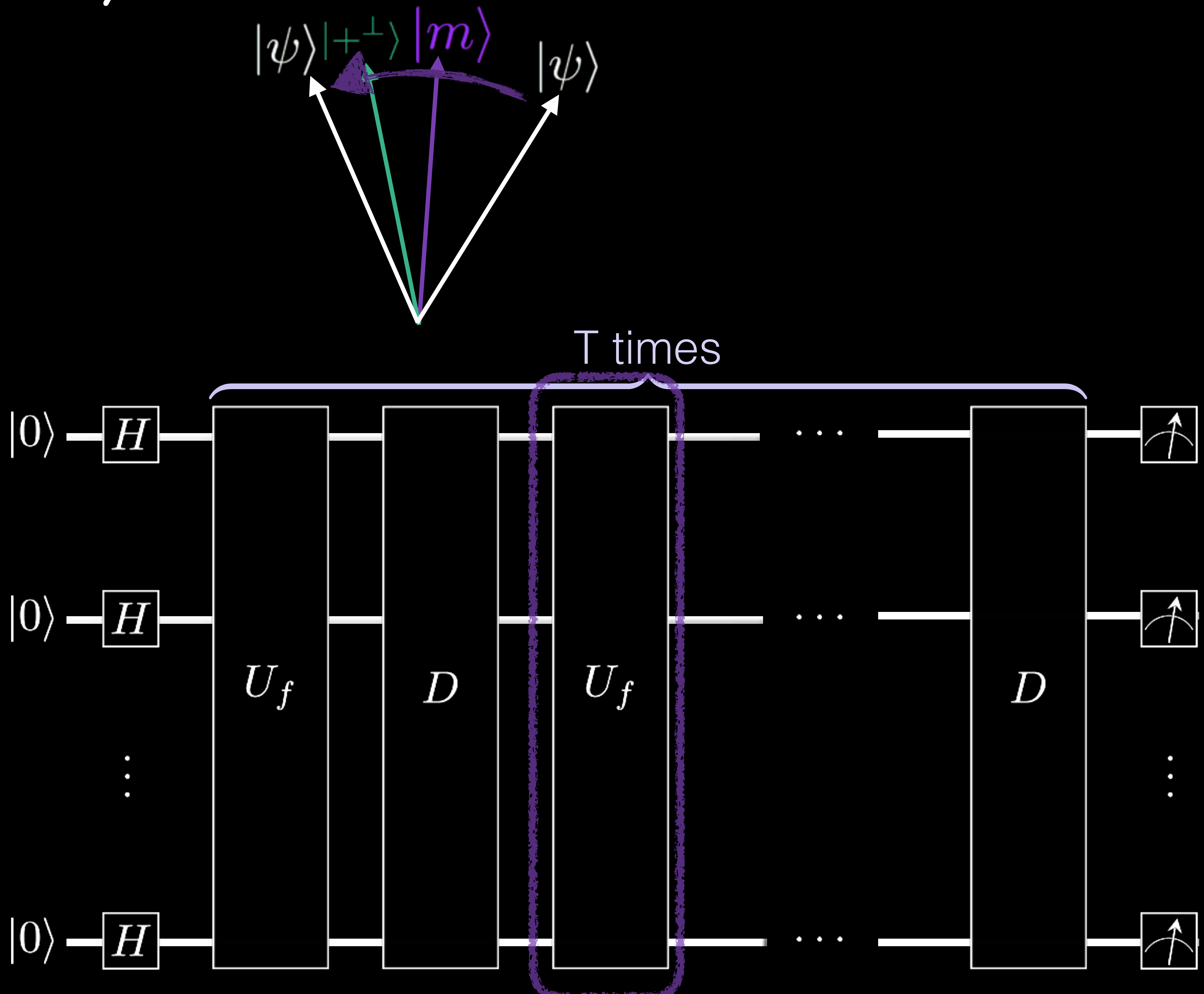

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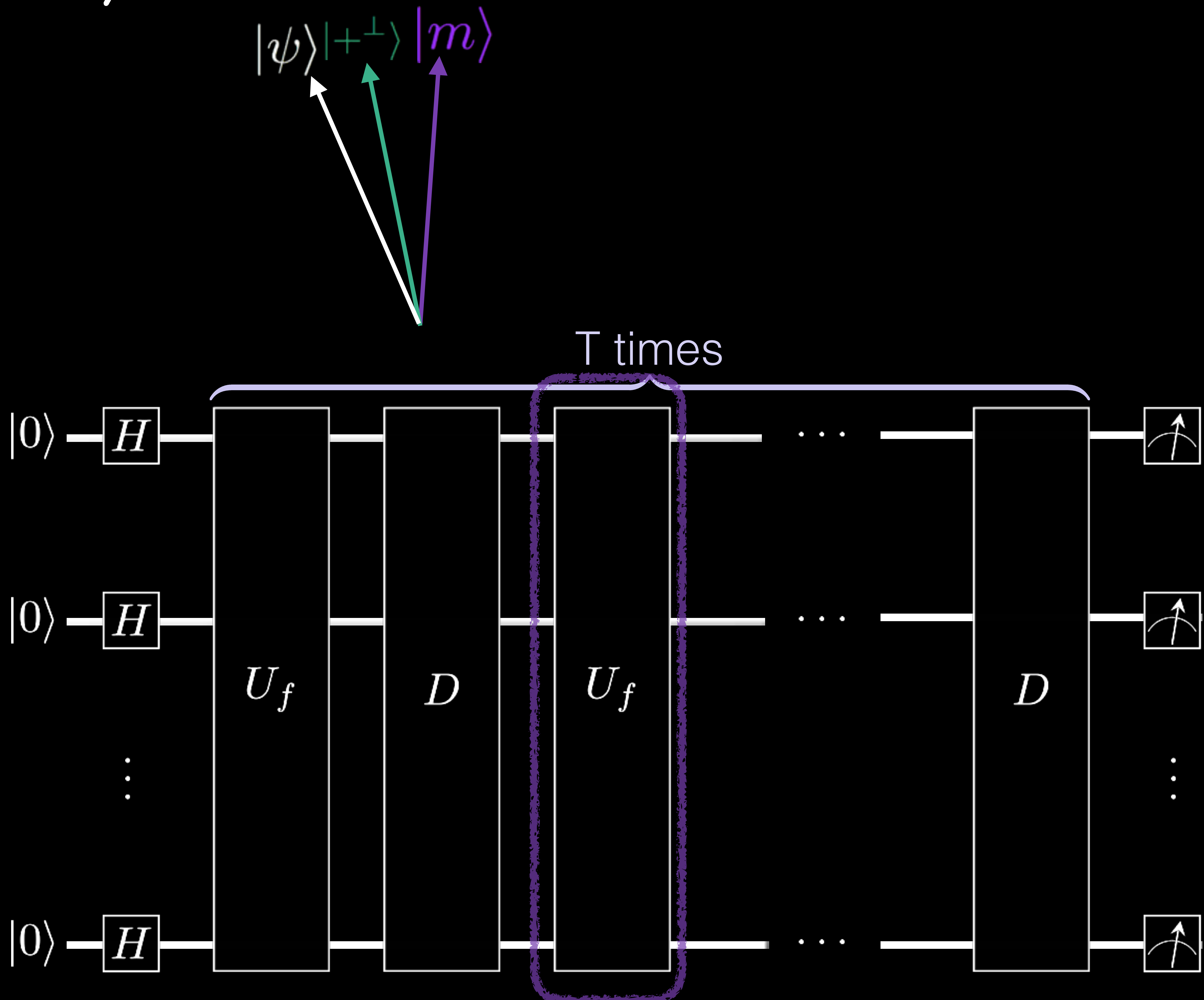

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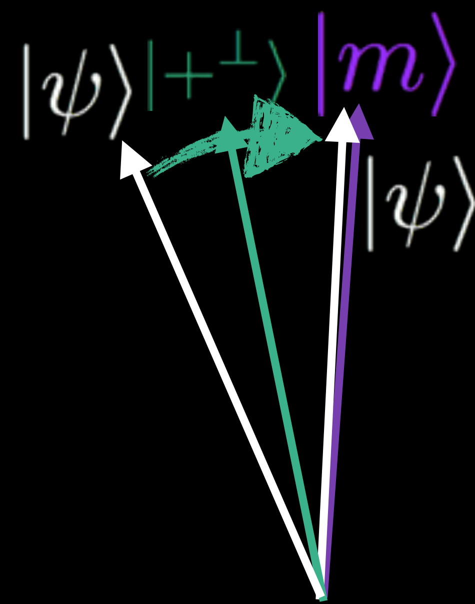

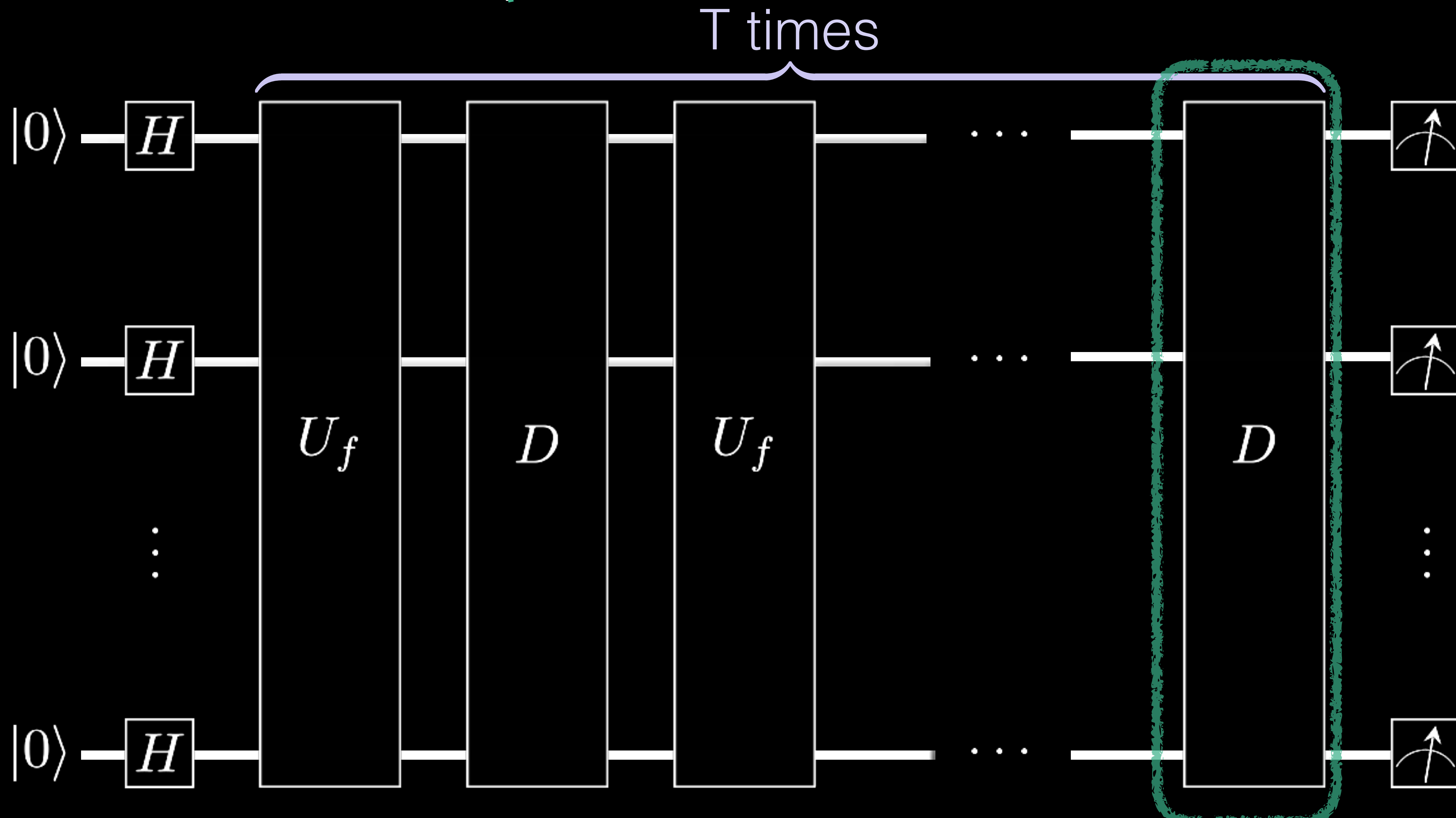

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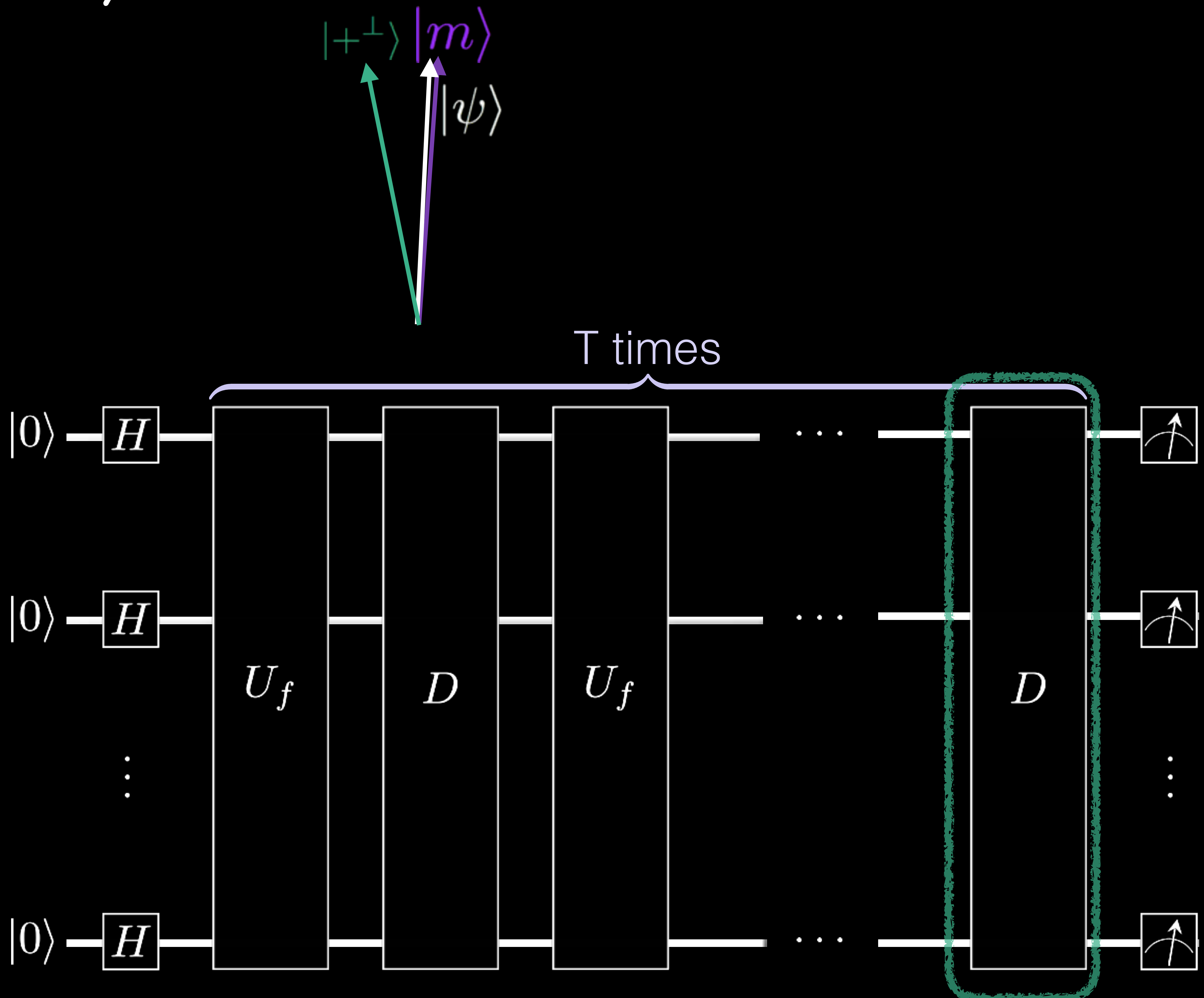

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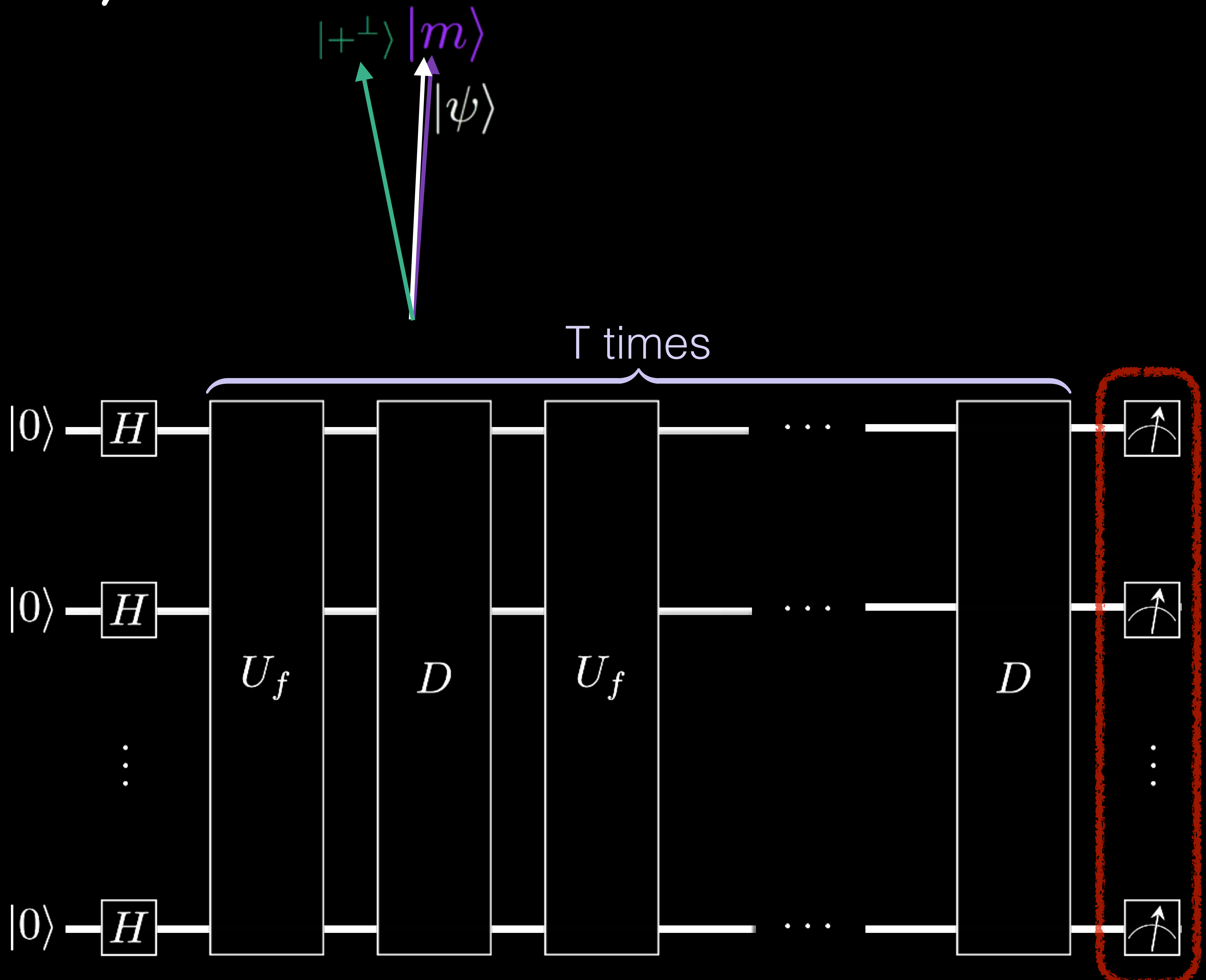

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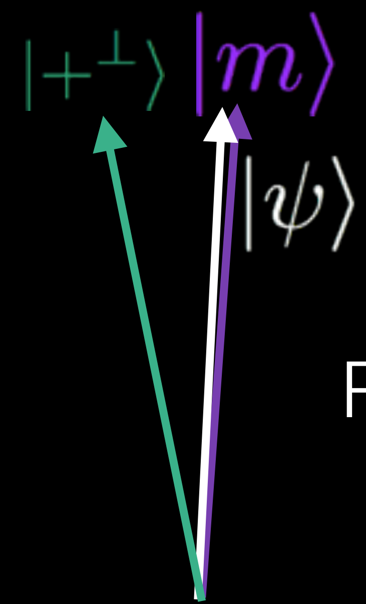
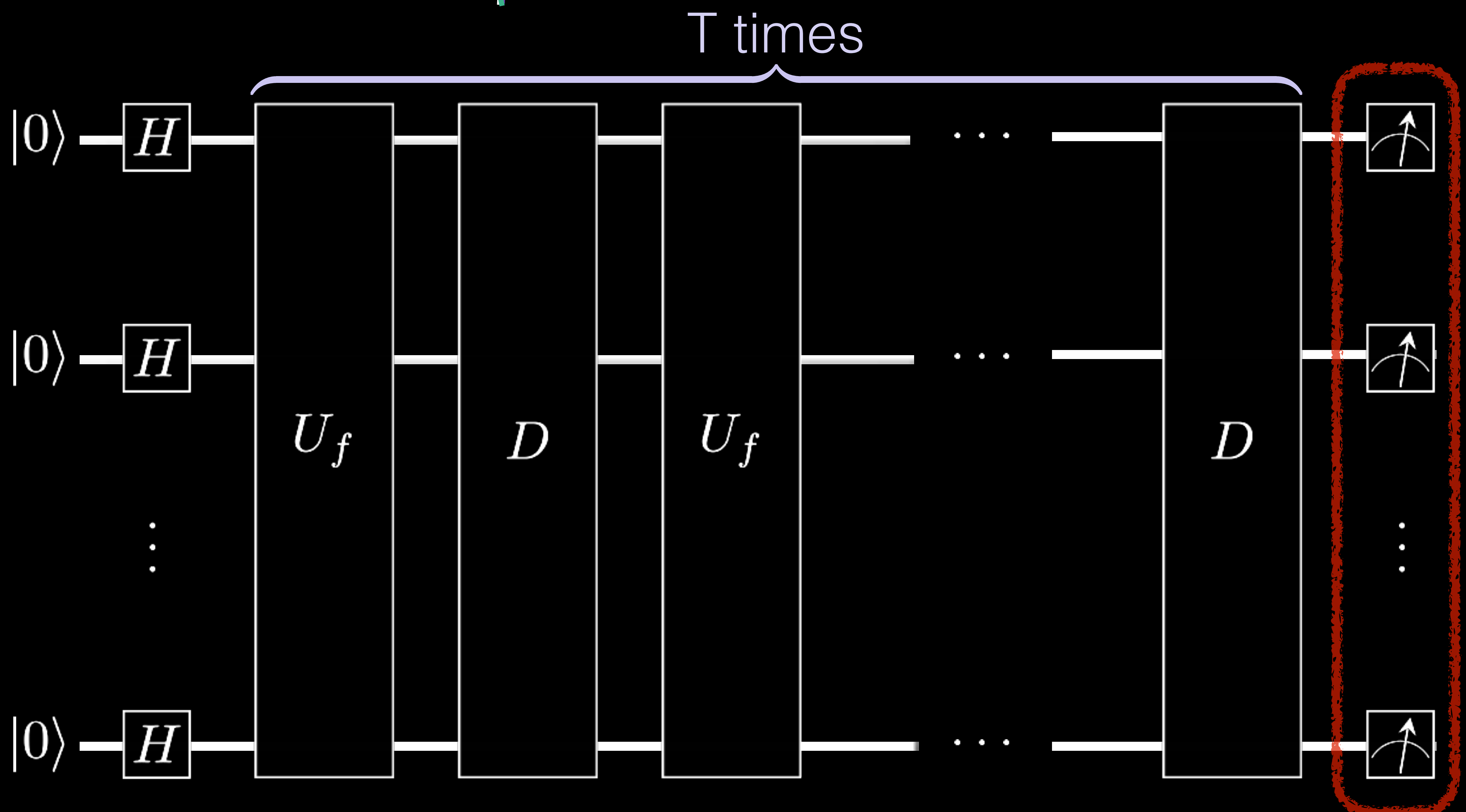


Probability of measuring $|m\rangle = |\langle m|\psi\rangle|^2$



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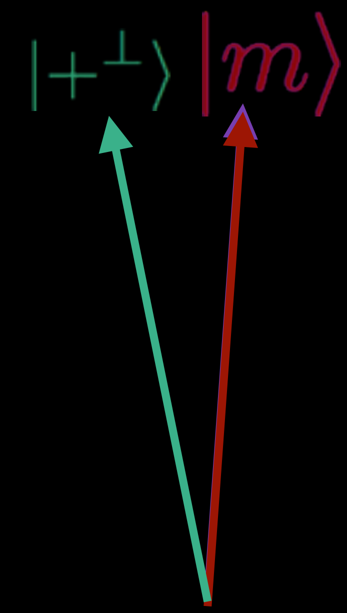
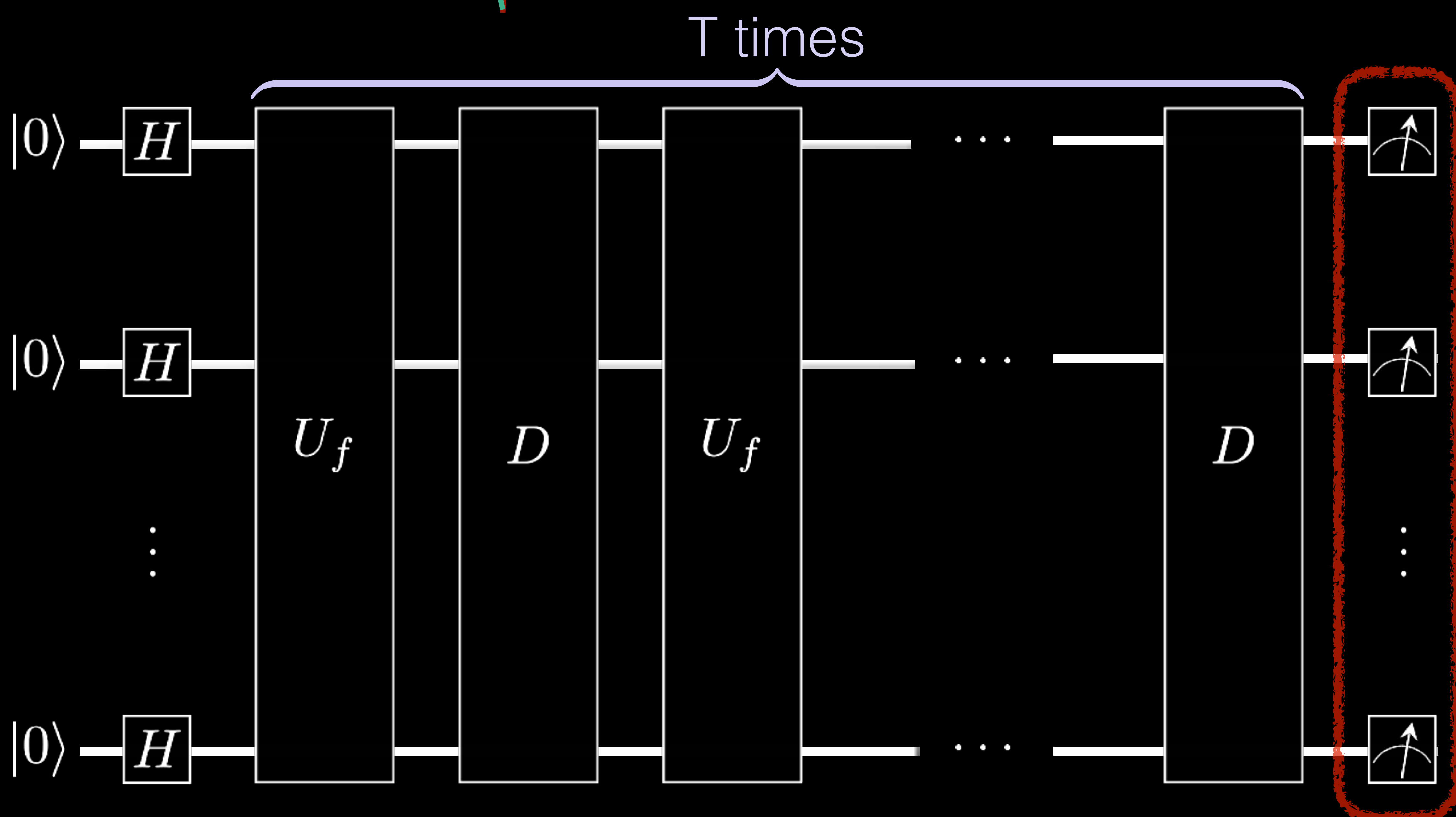


With high probability, the algorithm outputs **m**

Probability of measuring $|m\rangle = |\langle m|\psi\rangle|^2$



How long does it take?

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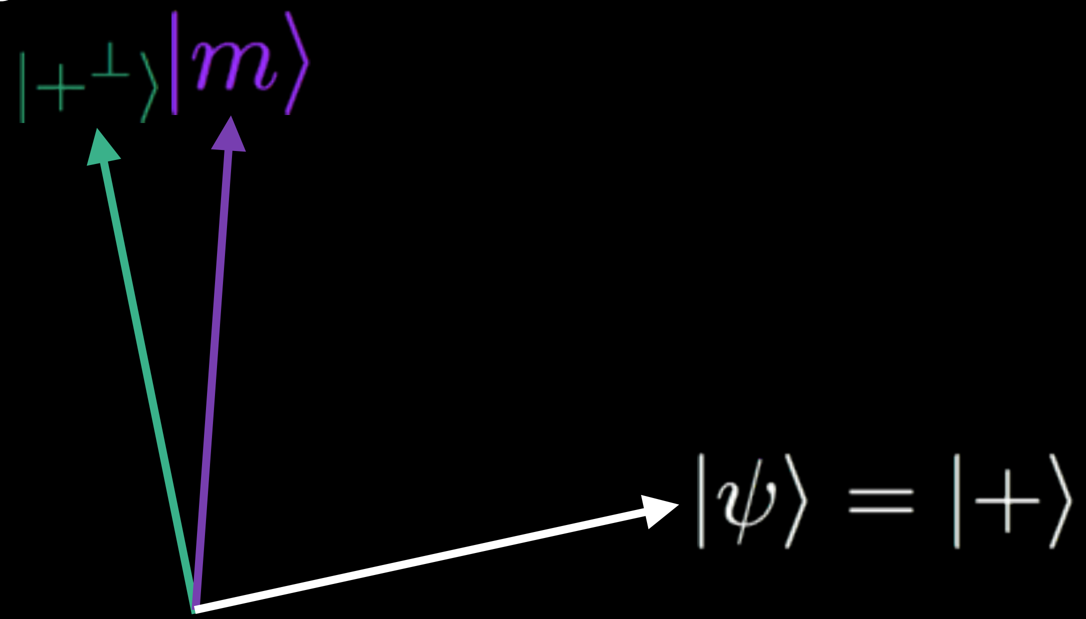

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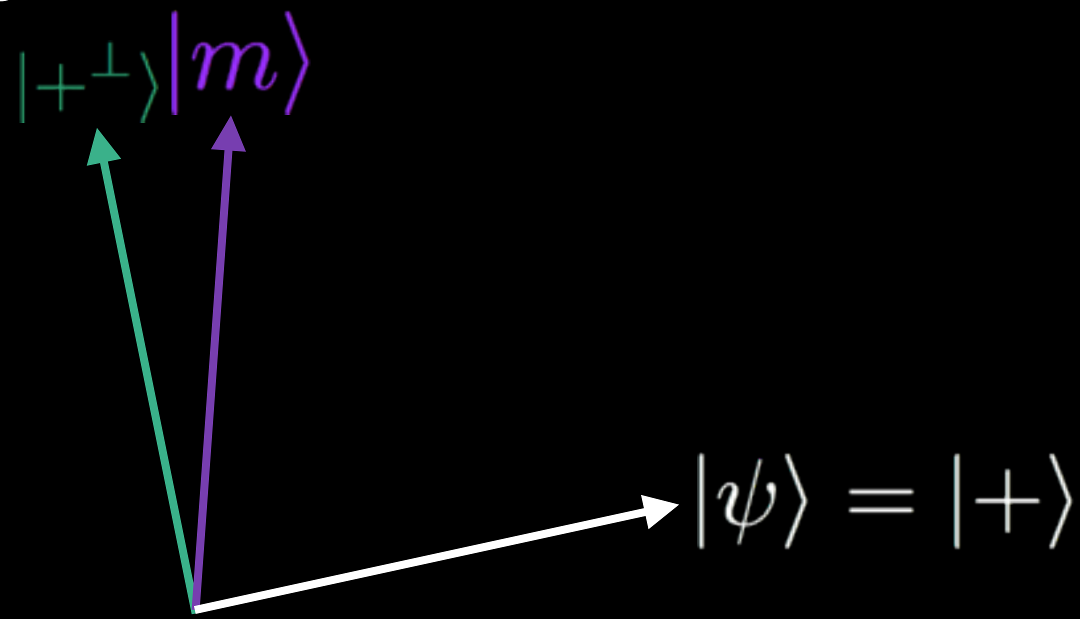
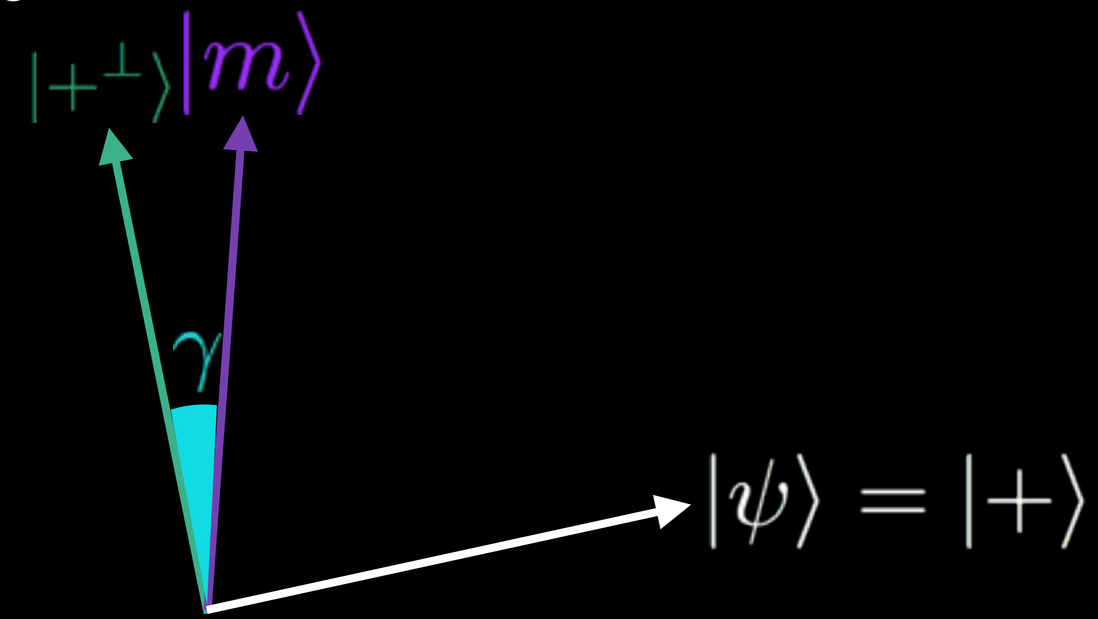


How much closer do we get after each iteration?

How long does it take?



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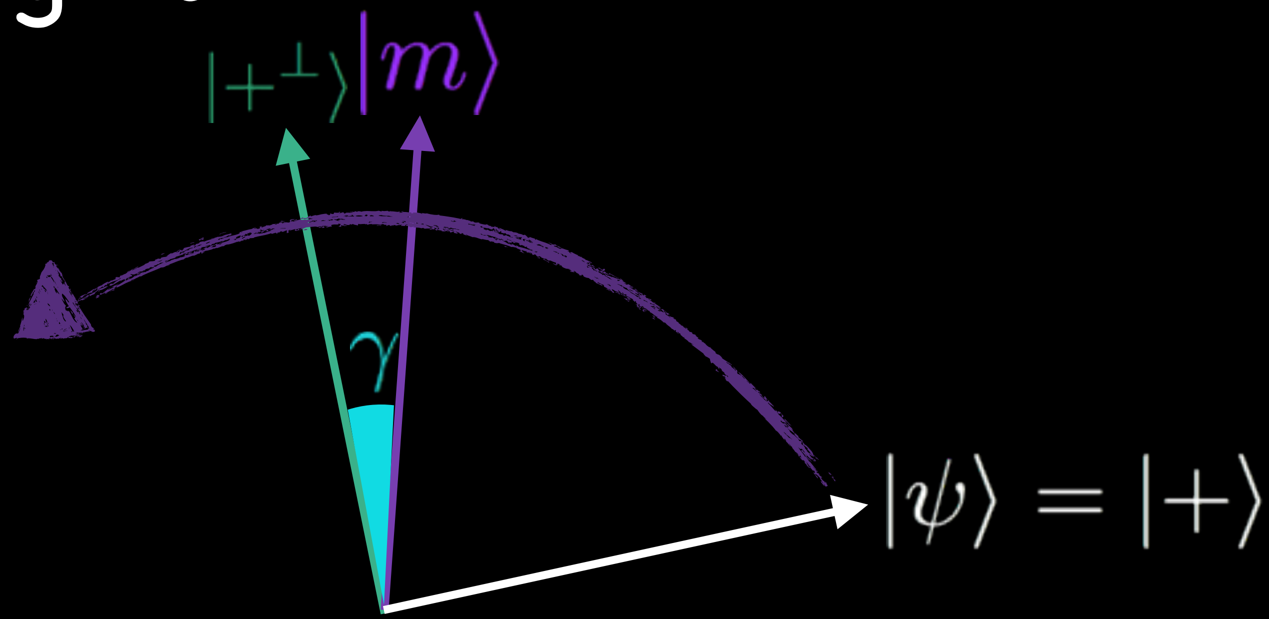

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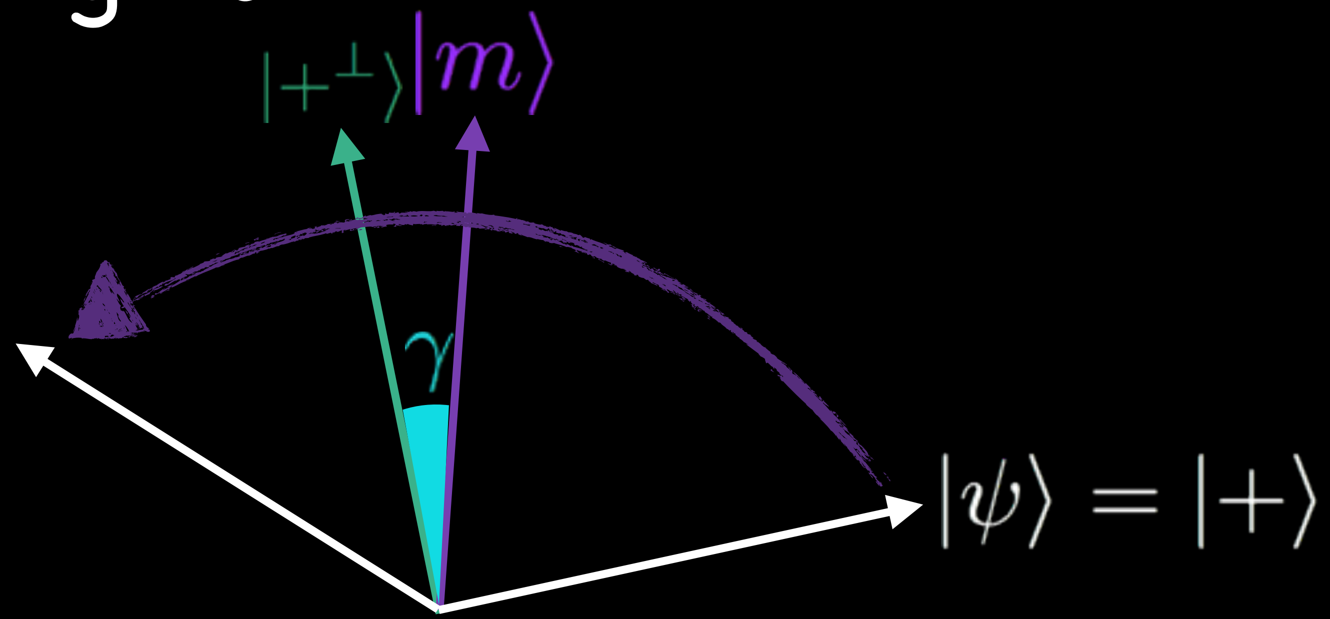

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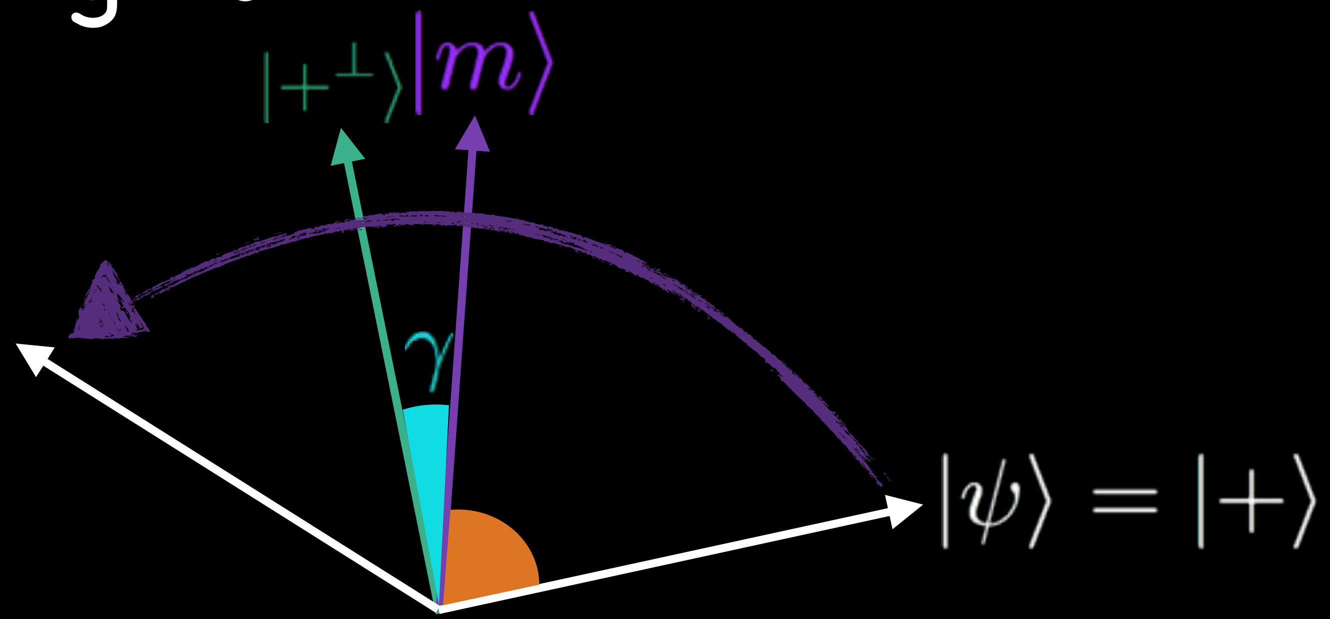

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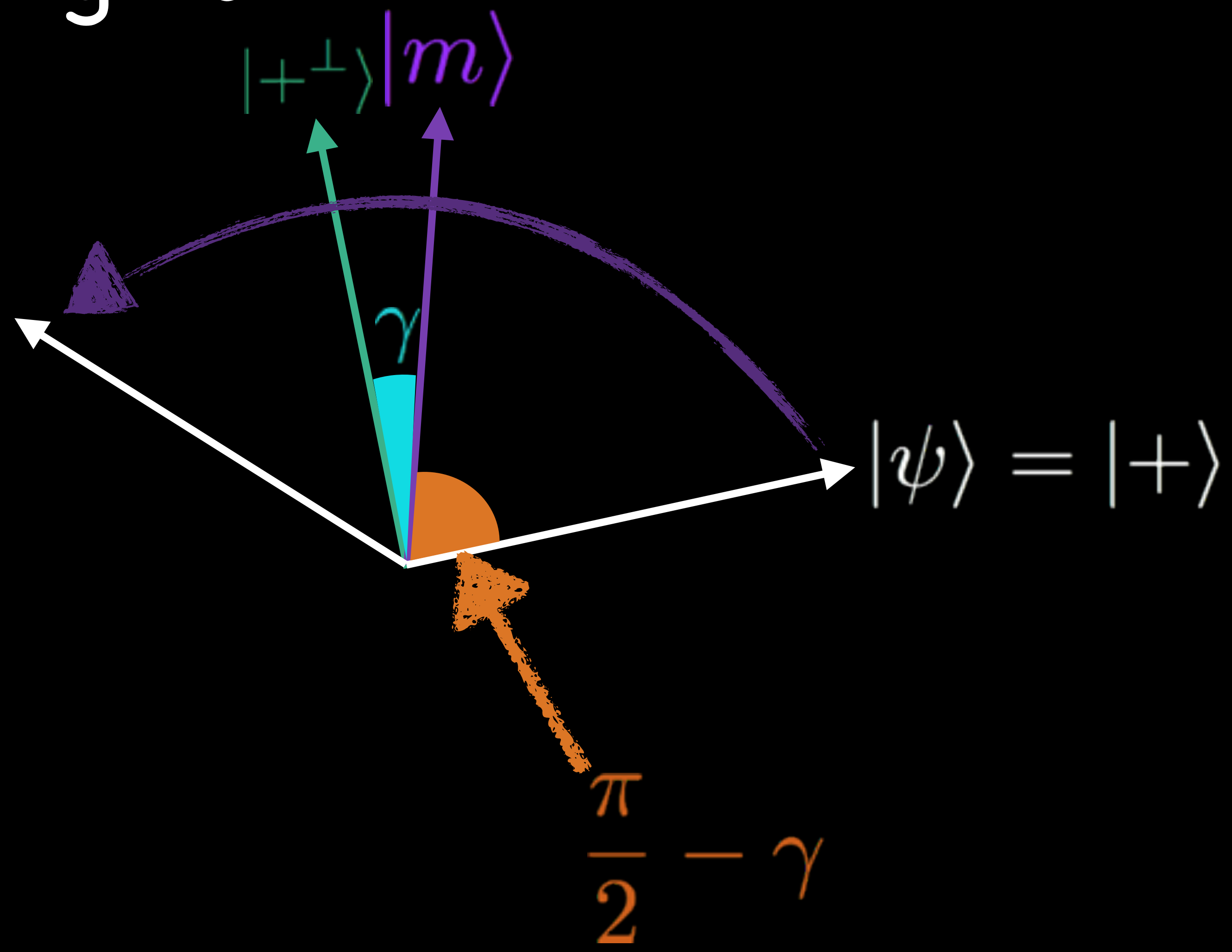

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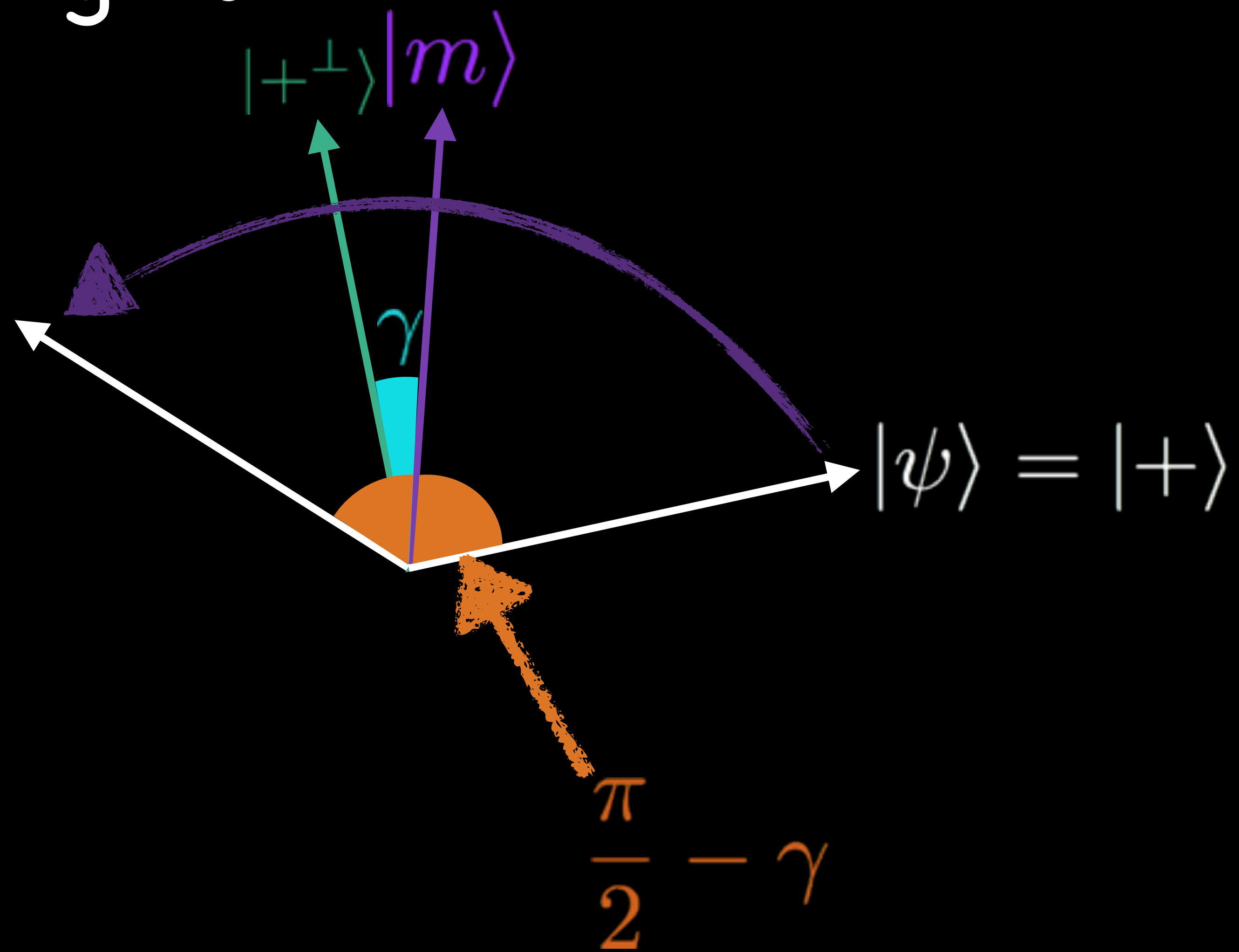

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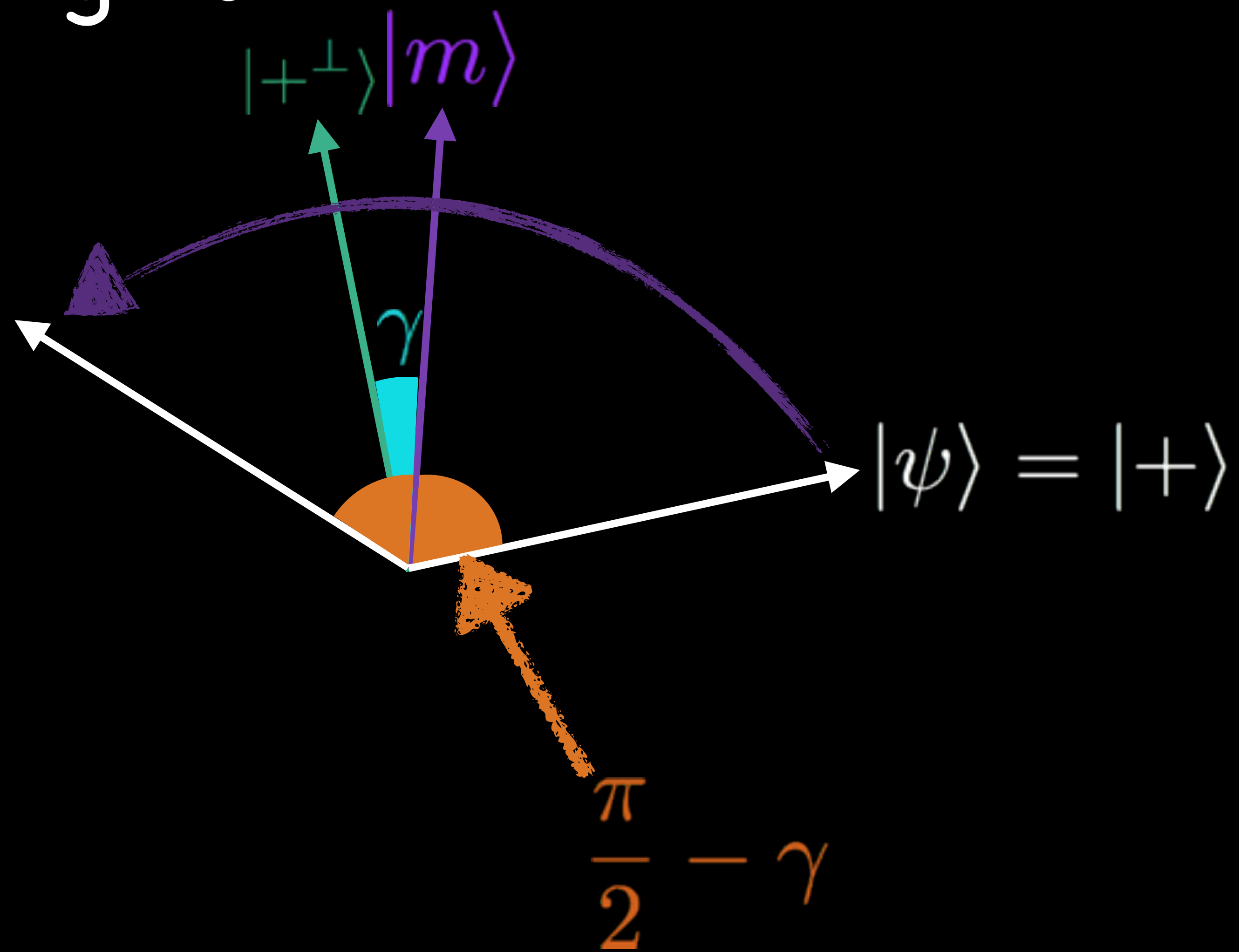
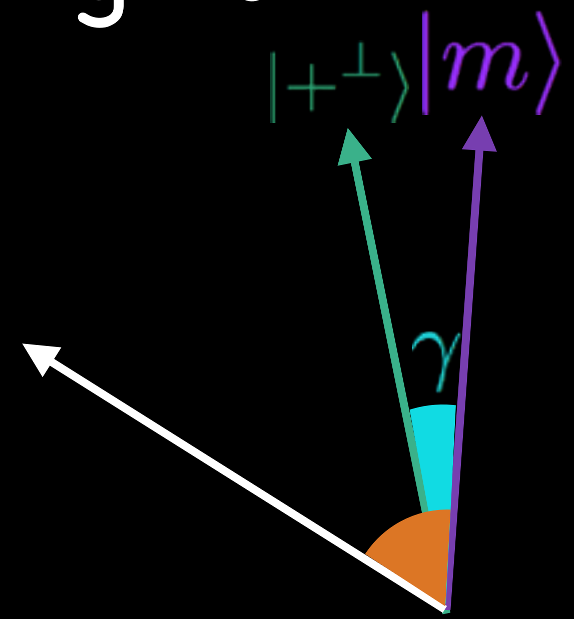


How much closer do we get after each iteration?

$$\left(\frac{\pi}{2} - \gamma\right) + \left(\frac{\pi}{2} - \gamma\right)$$

How long does it take?



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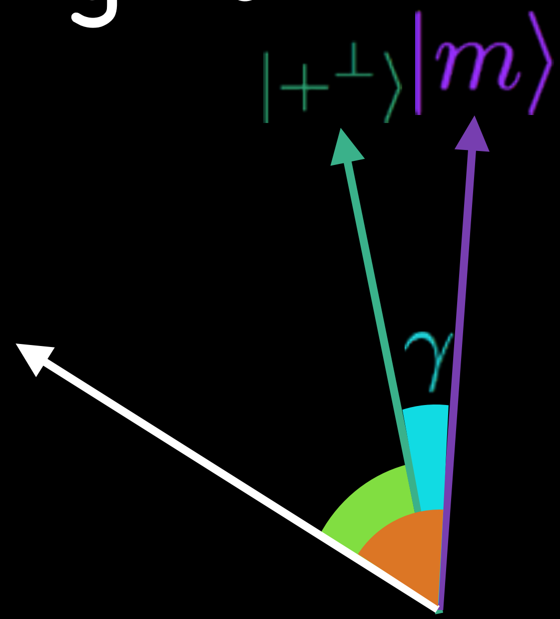

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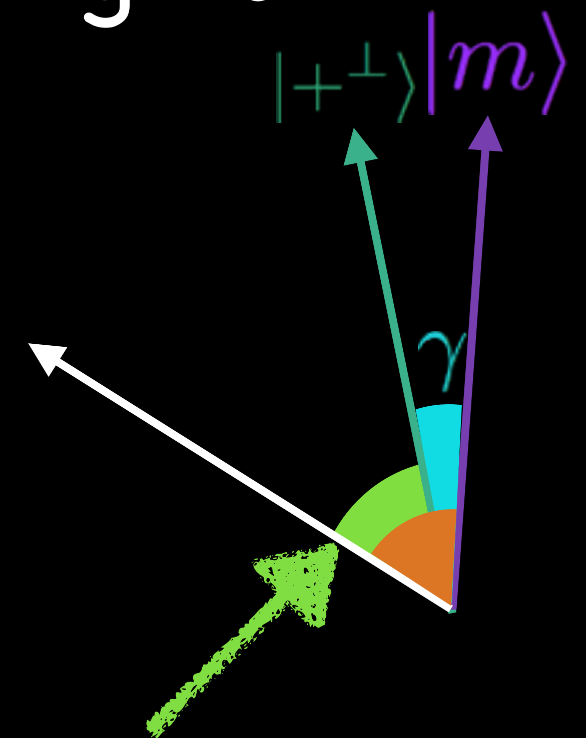
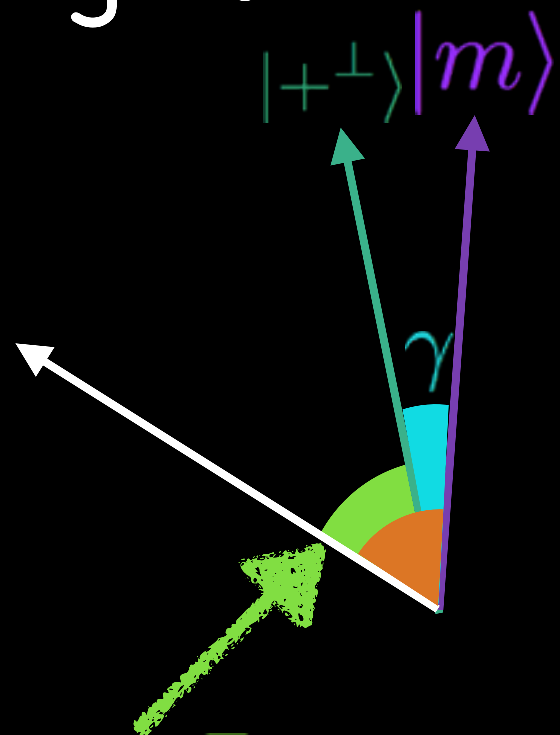


$$\left(\frac{\pi}{2} - \gamma\right) - \gamma$$

How much closer do we get after each iteration?

$$\left(\frac{\pi}{2} - \gamma\right) + \left(\frac{\pi}{2} - \gamma\right)$$

How long does it take?



$$\left(\frac{\pi}{2} - \gamma\right) - \gamma = \frac{\pi}{2} - 2\gamma$$

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$$\left(\frac{\pi}{2} - \gamma\right) + \left(\frac{\pi}{2} - \gamma\right)$$

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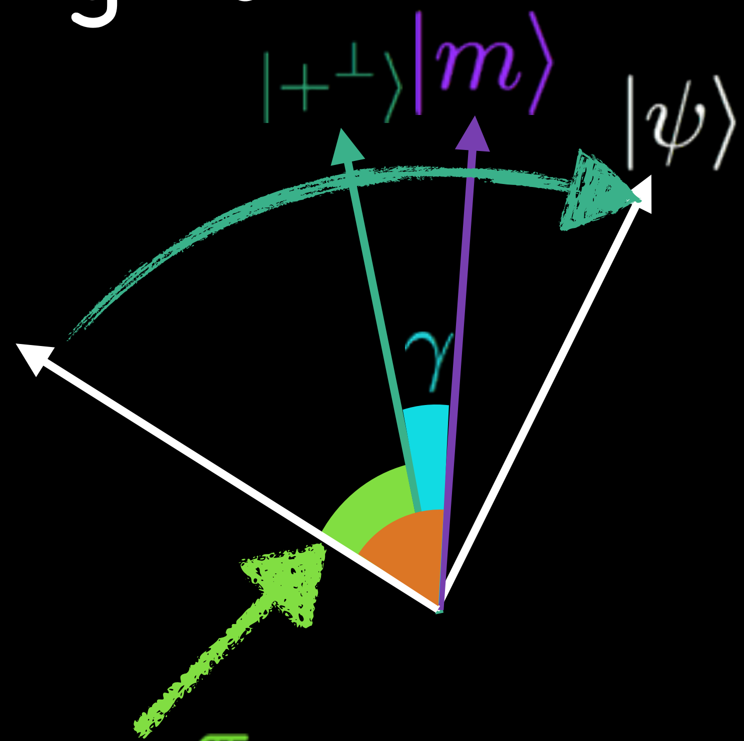

$$\left(\frac{\pi}{2}-\gamma\right)-\gamma=\frac{\pi}{2}-2\gamma$$

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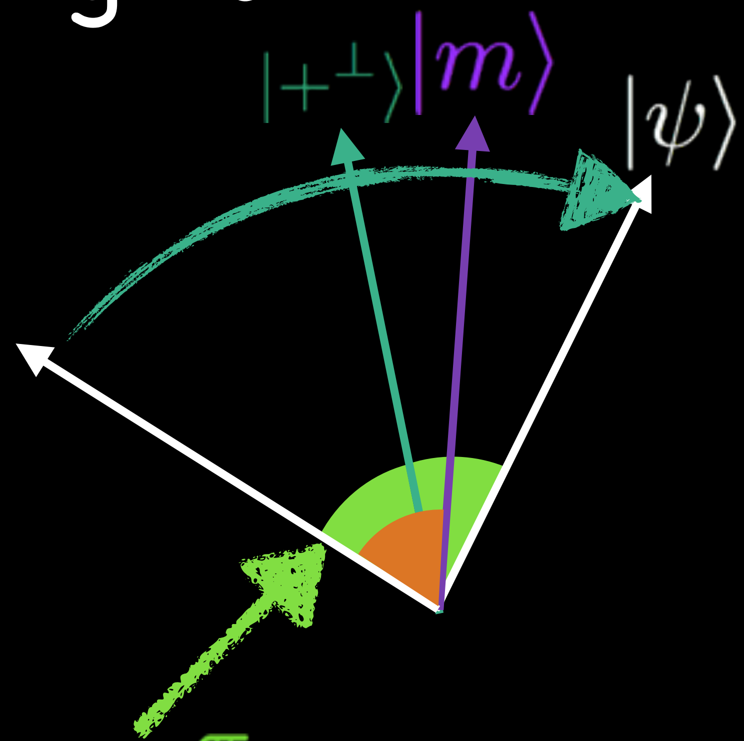
$$\left(\frac{\pi}{2}-\gamma\right)+\left(\frac{\pi}{2}-\gamma\right)$$

How long does it take?



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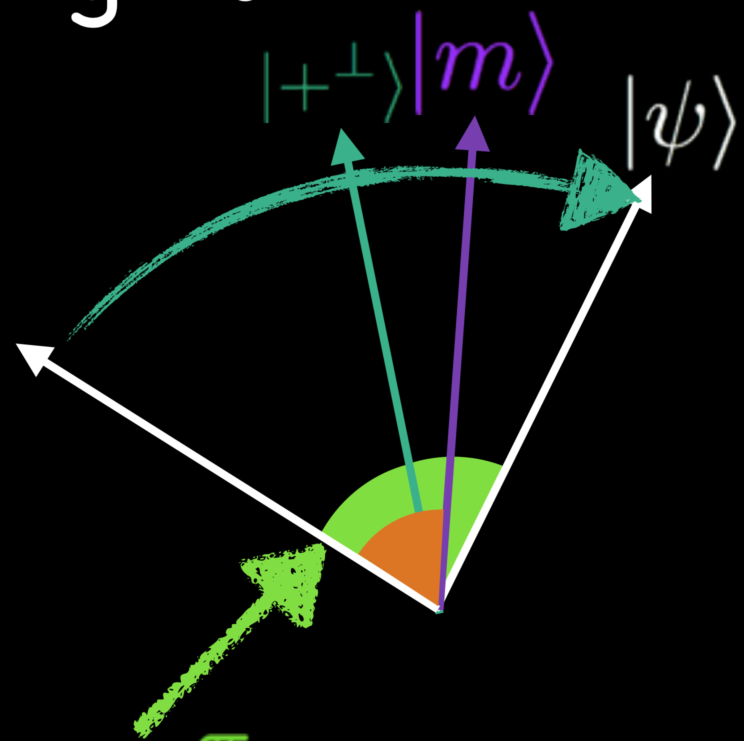
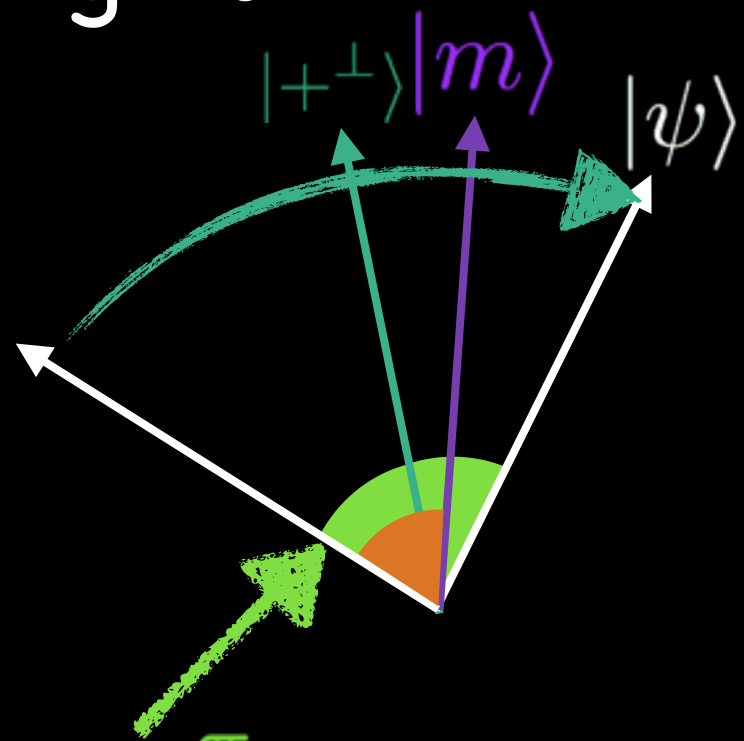


$$\left(\frac{\pi}{2} - \gamma\right) - \gamma = \frac{\pi}{2} - 2\gamma$$

How much closer do we get after each iteration?

$$\left(\frac{\pi}{2} - \gamma\right) + \left(\frac{\pi}{2} - \gamma\right) - \left(\frac{\pi}{2} - 2\gamma\right) - \left(\frac{\pi}{2} - 2\gamma\right)$$

How long does it take?

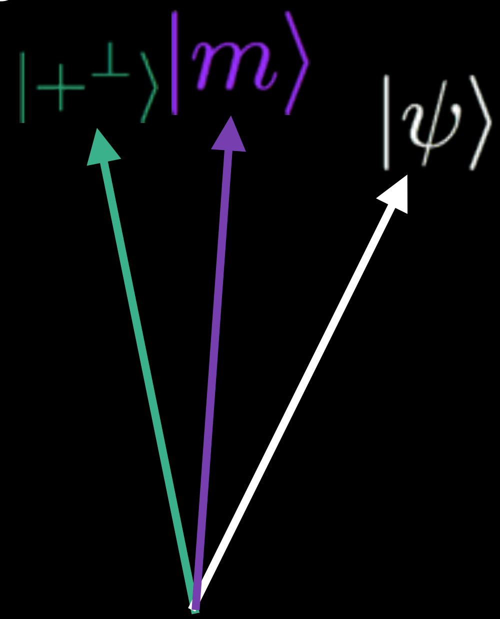


$$\left(\frac{\pi}{2}-\gamma\right)-\gamma=\frac{\pi}{2}-2\gamma$$

How much closer do we get after each iteration?

$$\left(\frac{\pi}{2}-\gamma\right)+\left(\frac{\pi}{2}-\gamma\right)-\left(\frac{\pi}{2}-2\gamma\right)-\left(\frac{\pi}{2}-2\gamma\right)$$
$$=2\gamma$$

How long does it take?



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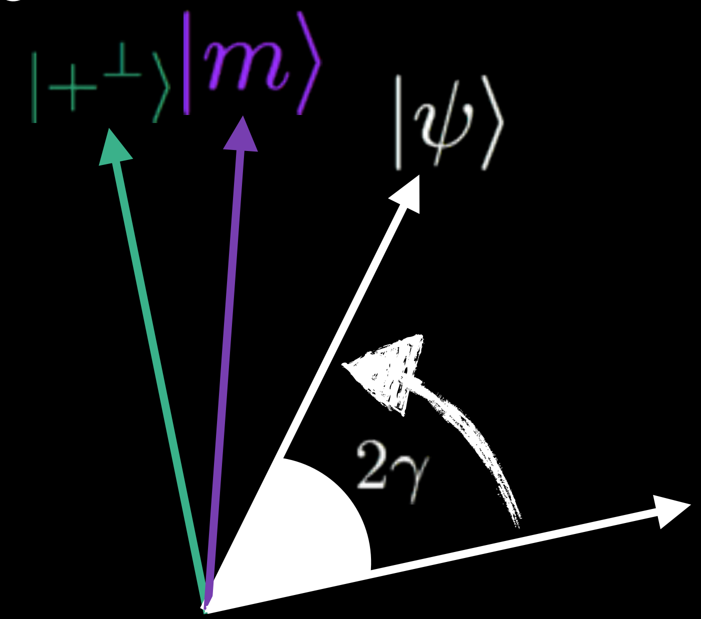

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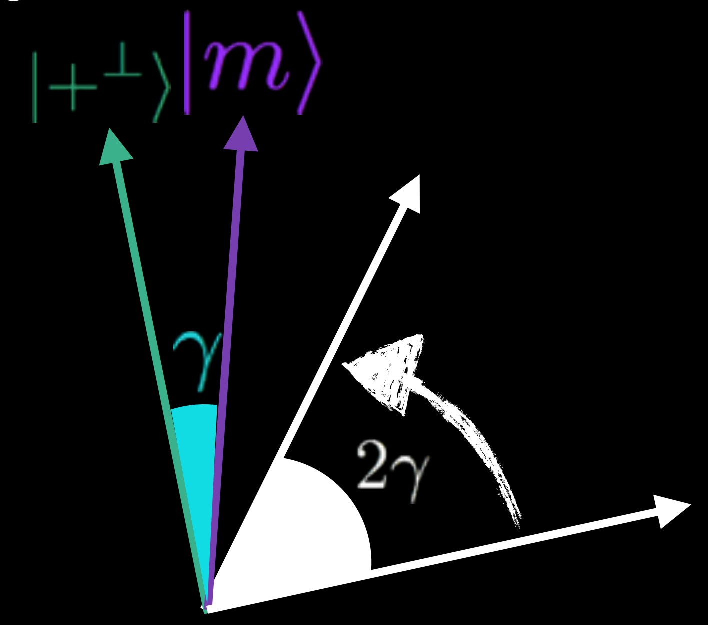

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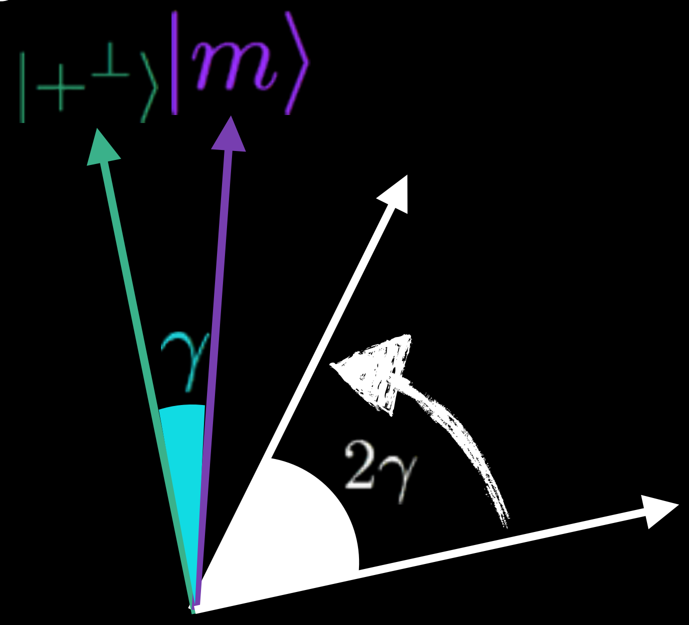

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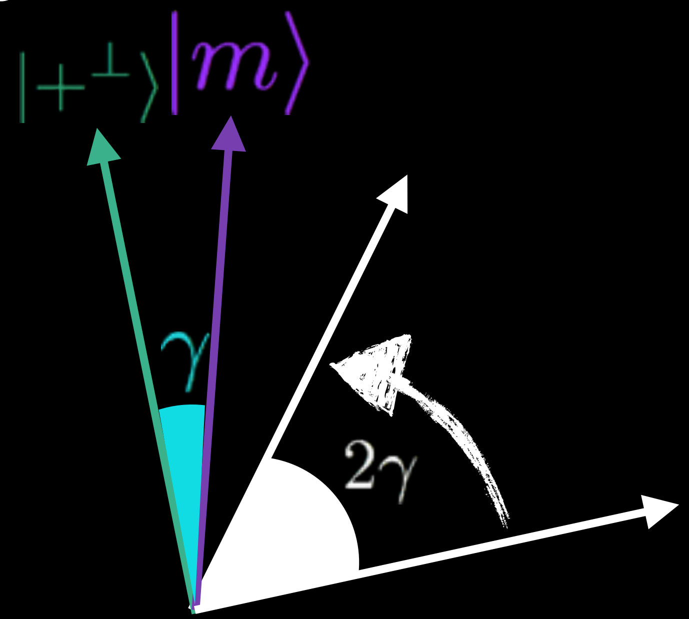

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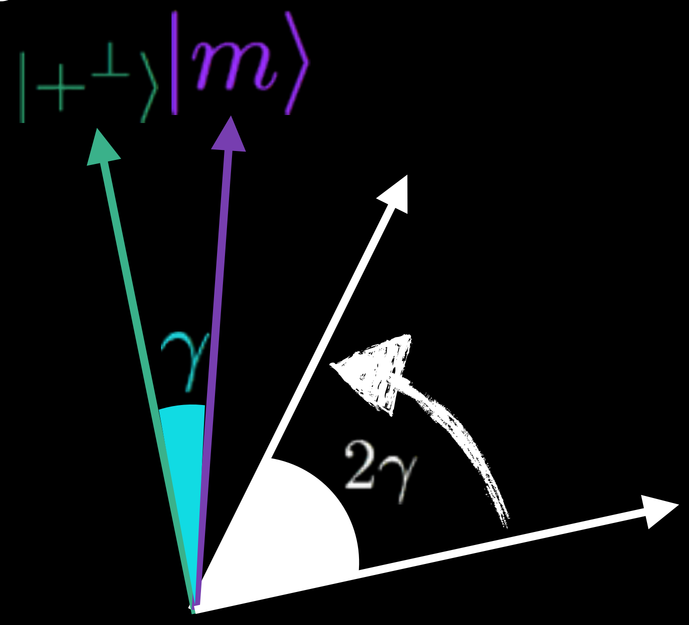

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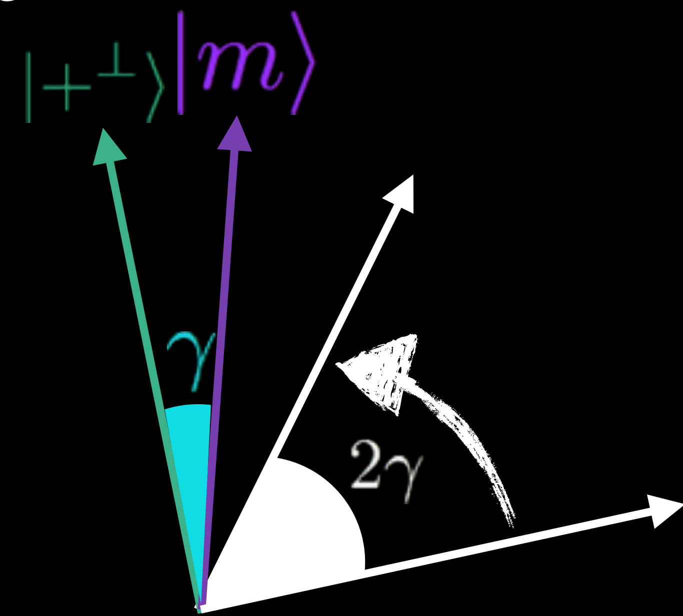

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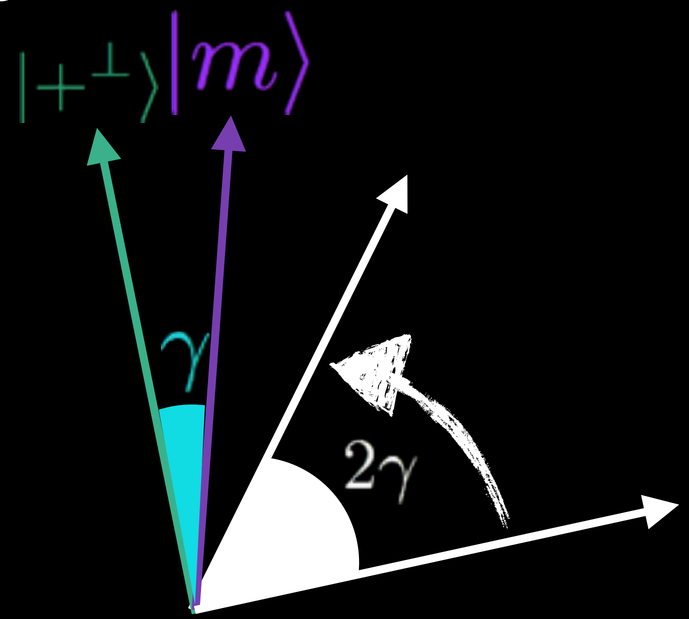

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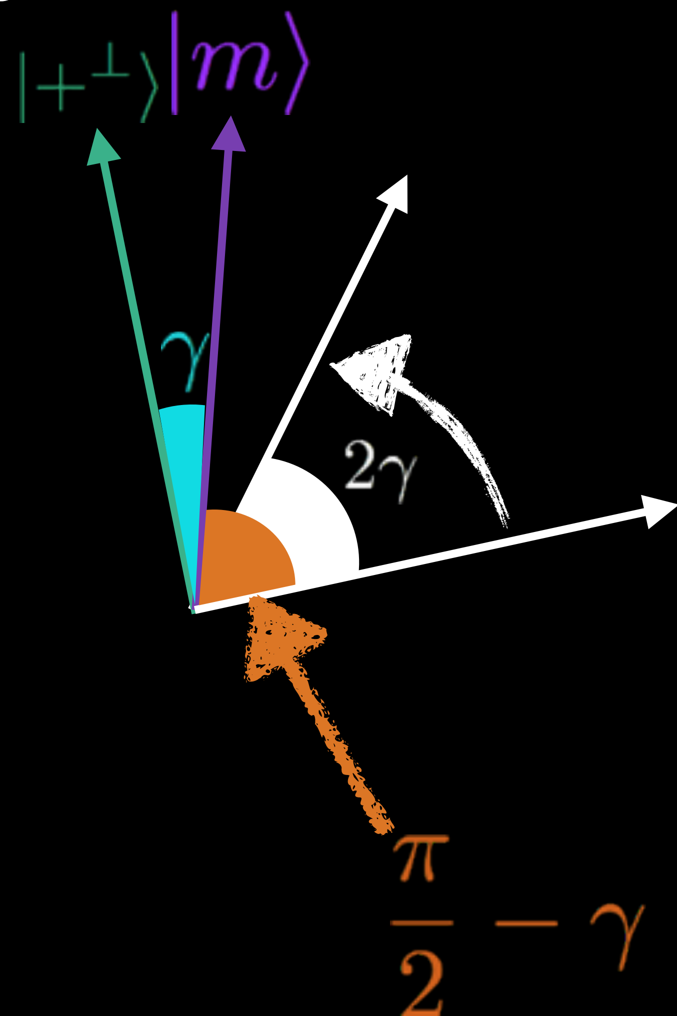

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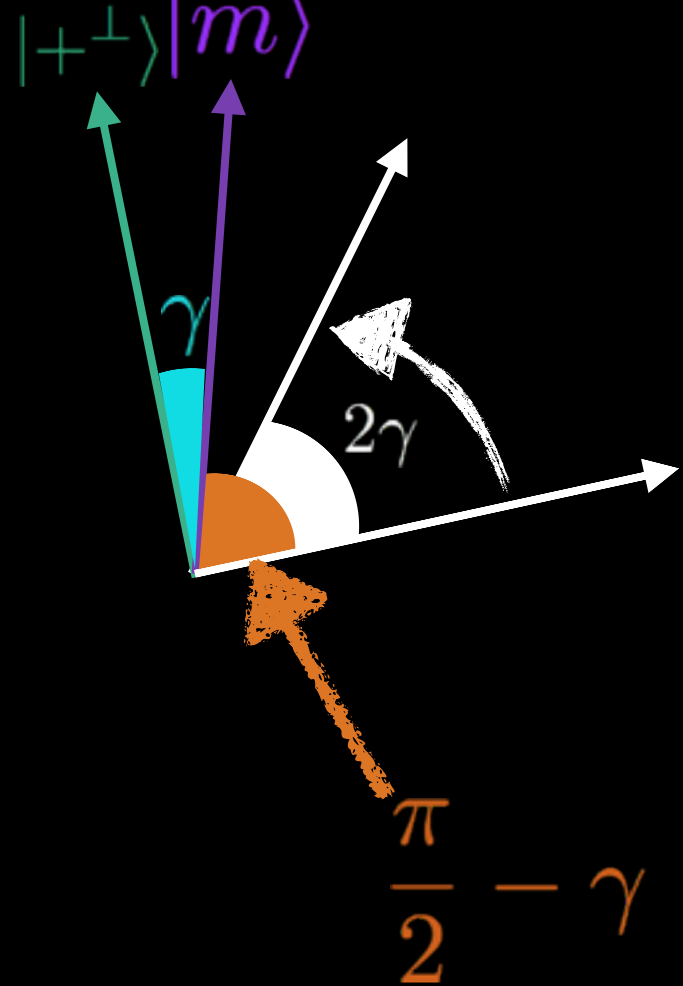

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$$\frac{\pi}{2} - \gamma$$

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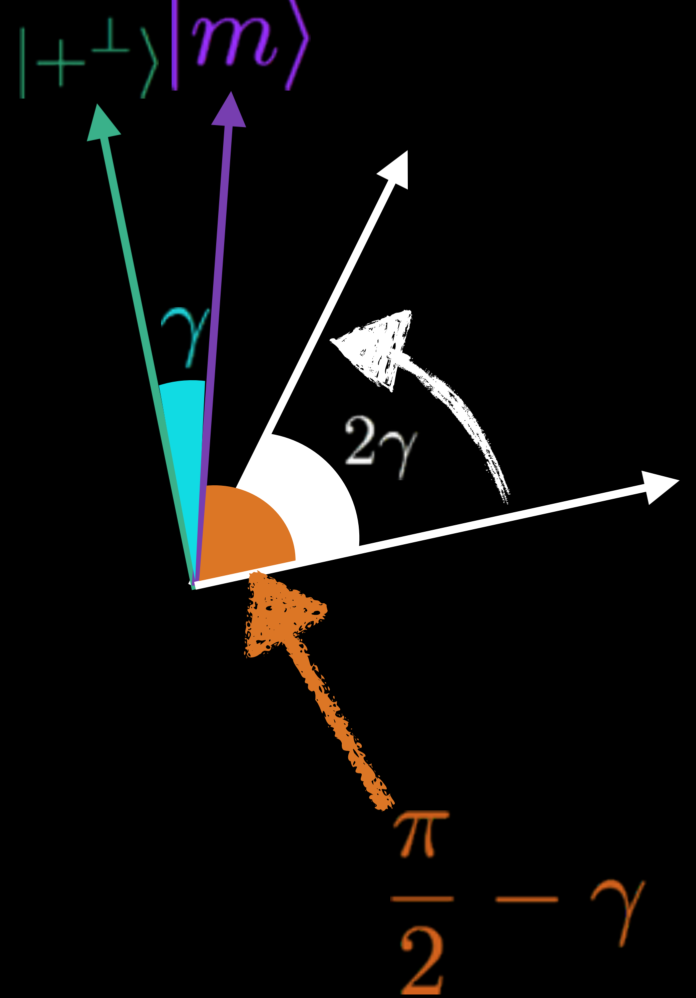

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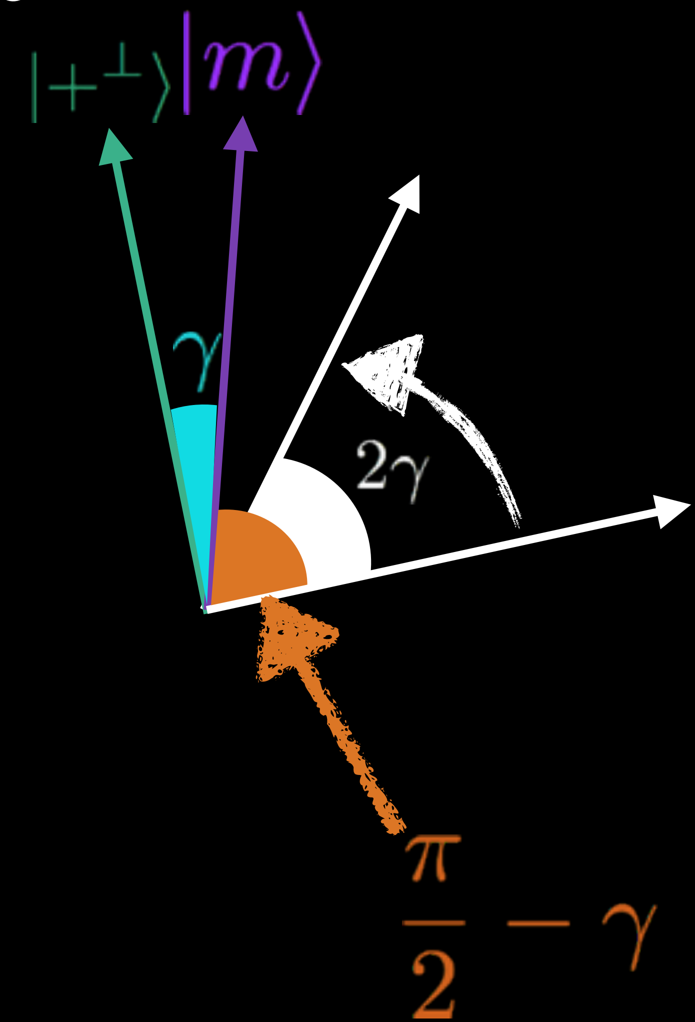

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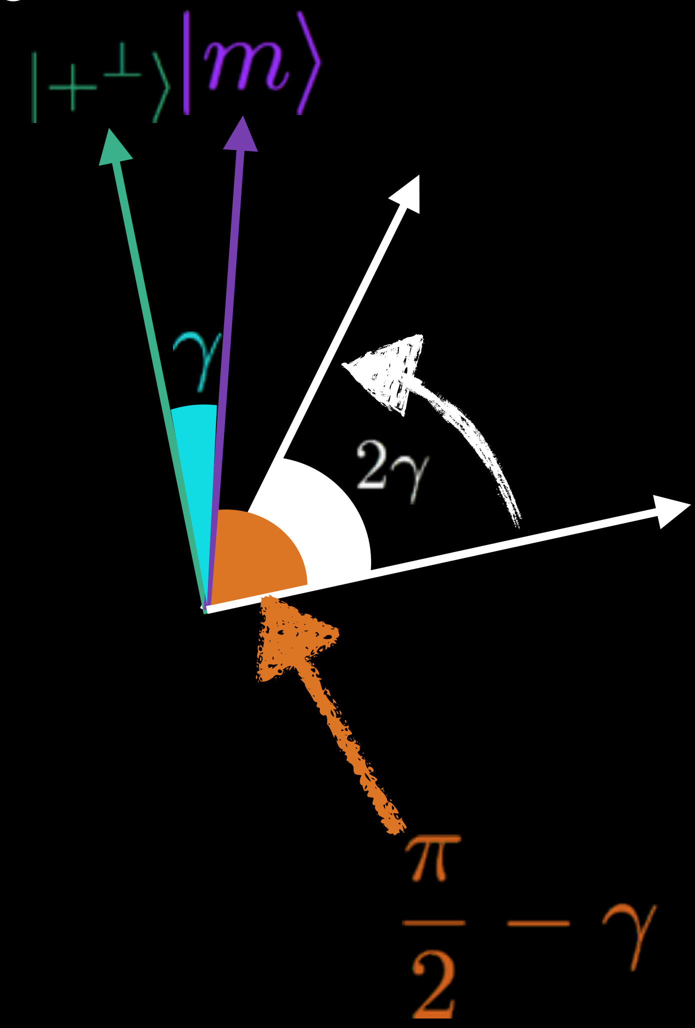

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- Gives a generic square-root speedup
    - But not always faster than classical!

Phase Estimation

Phase Estimation

- An important quantum computing primitive

Phase Estimation

- An important quantum computing primitive
- Often used as an ingredient in more complex algorithms:
    - Integer factorisation
    - Matrix inversion
    - Quantum counting
    - Quantum walks
    - …

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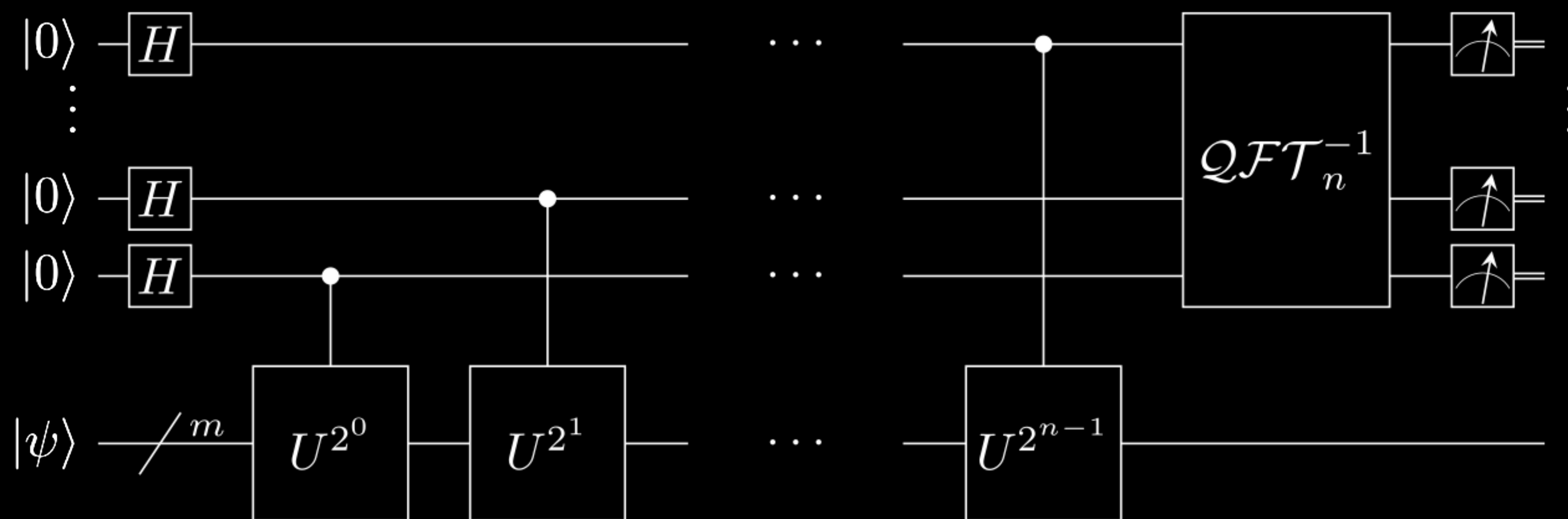
Phase Estimation: Given U and $|x\rangle$, estimate θ

Phase Estimation Circuit

Phase Estimation Circuit



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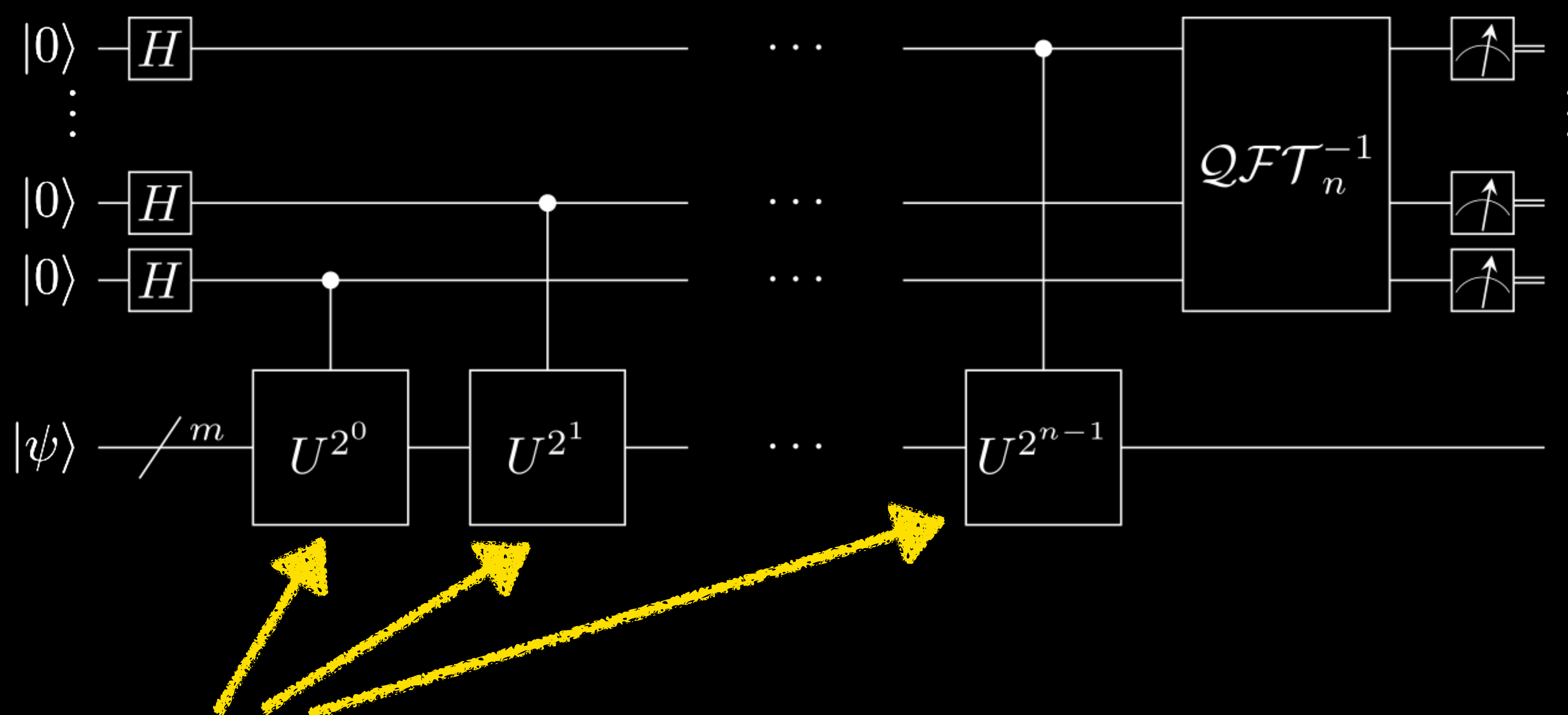

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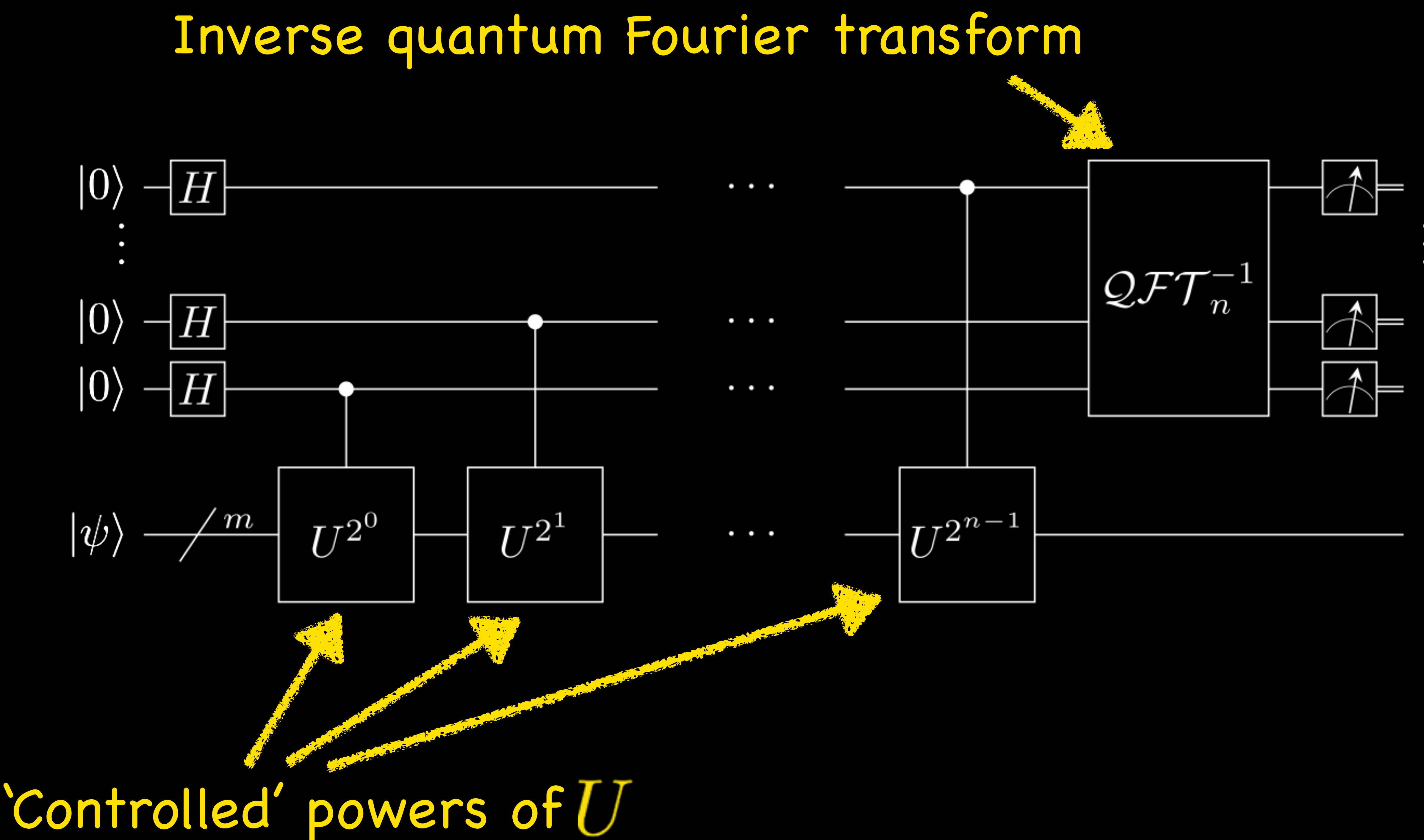
Phase Estimation Circuit



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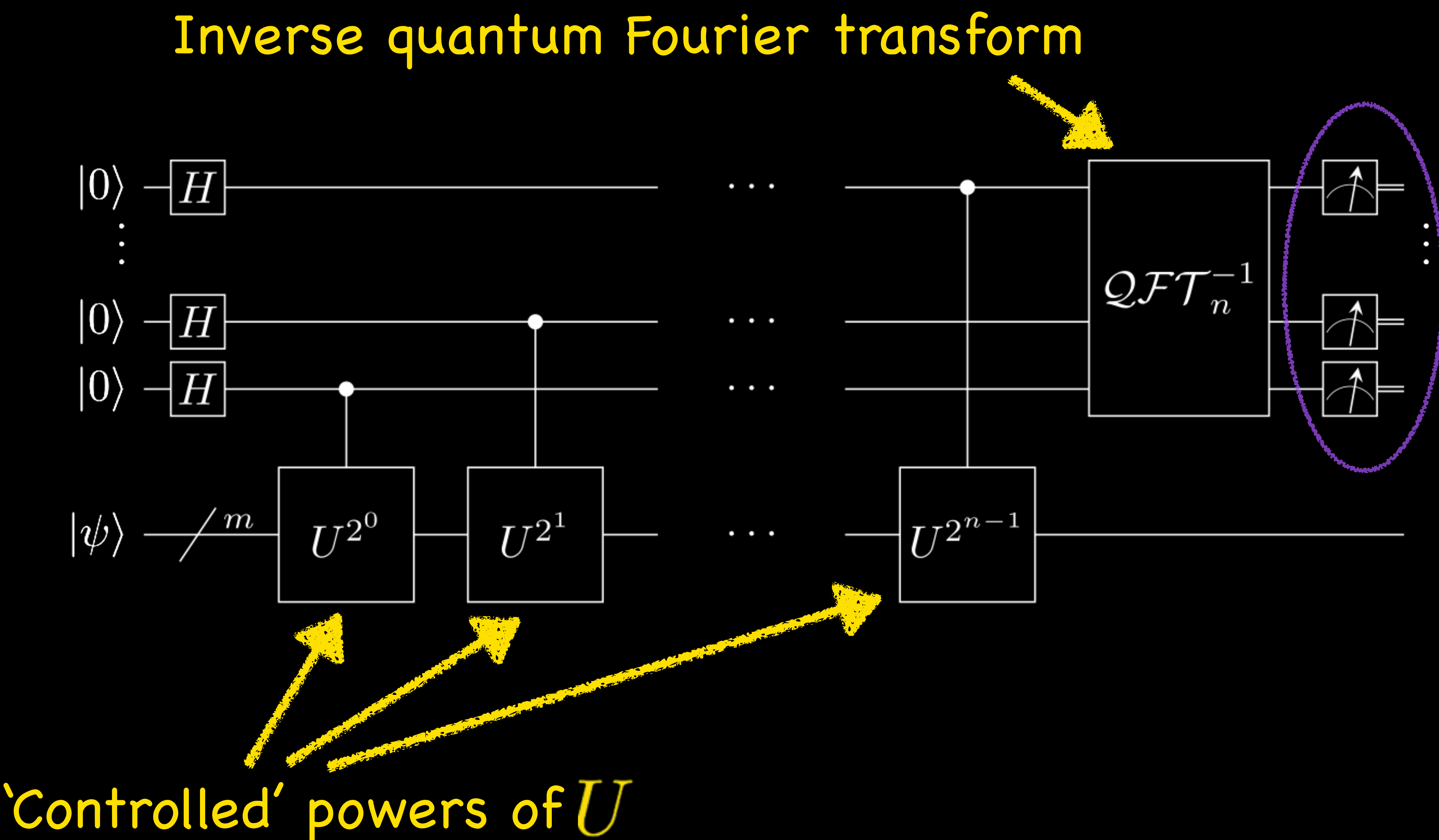
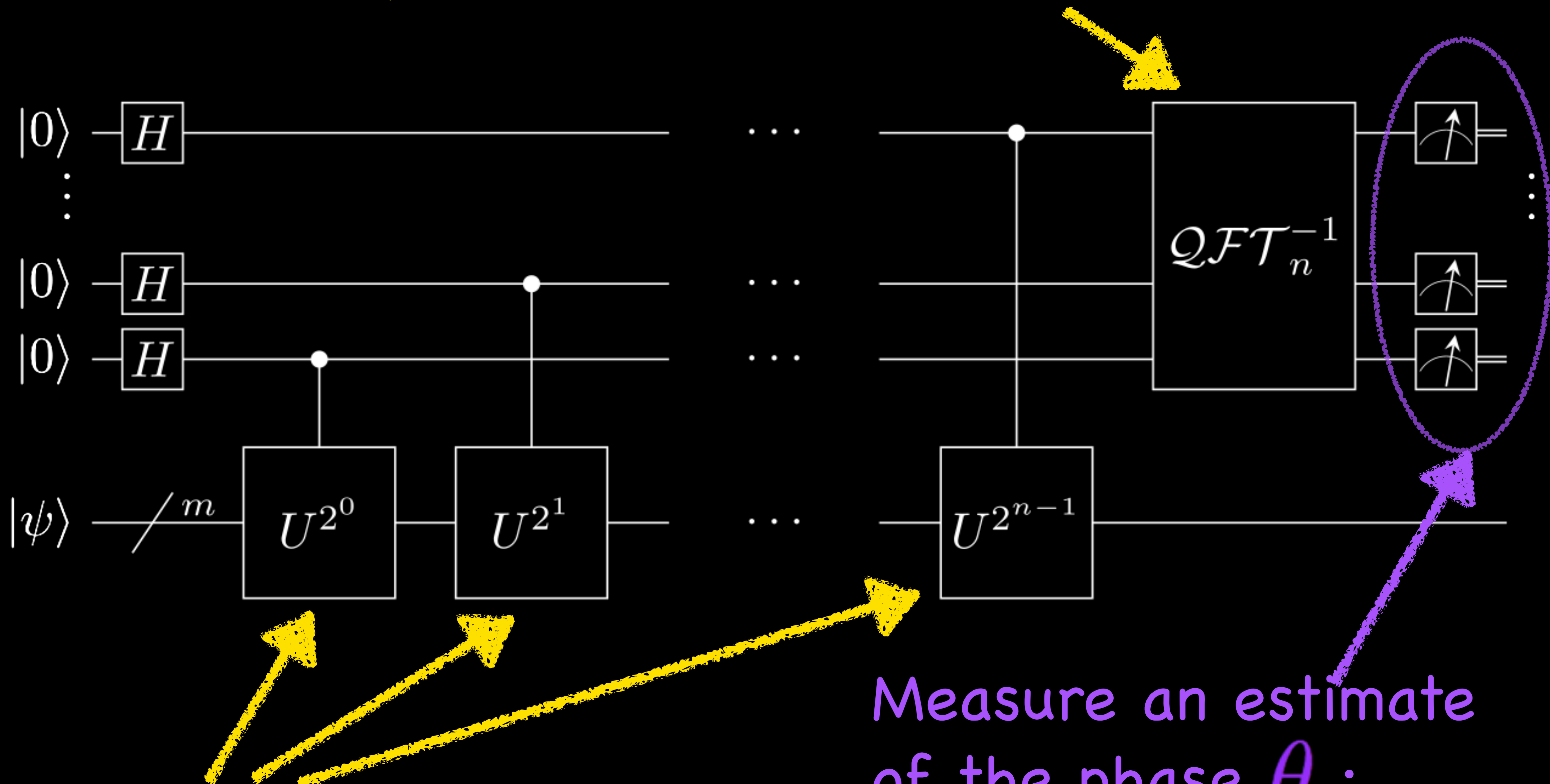


Phase Estimation Circuit

Inverse quantum Fourier transform



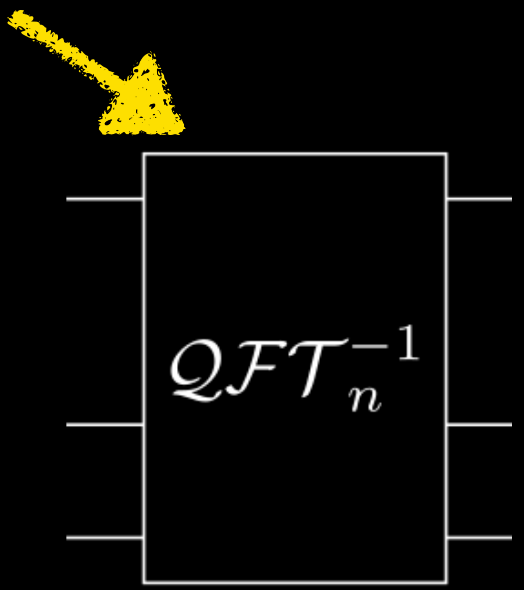
'Controlled' powers of U

Measure an estimate of the phase θ :

$$U|\psi\rangle = e^{2\pi i\theta}|\psi\rangle$$

Phase Estimation Circuit

Inverse quantum Fourier transform



Phase Estimation Circuit

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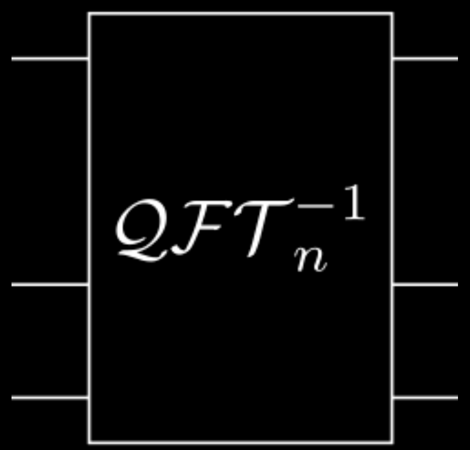

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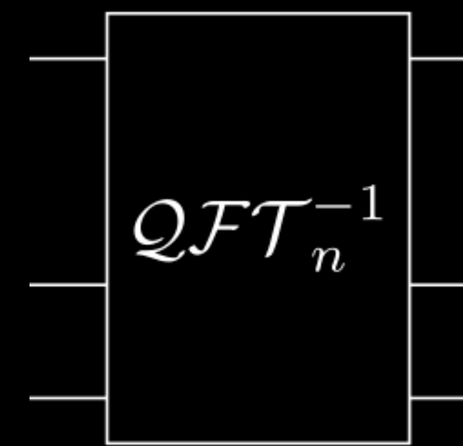

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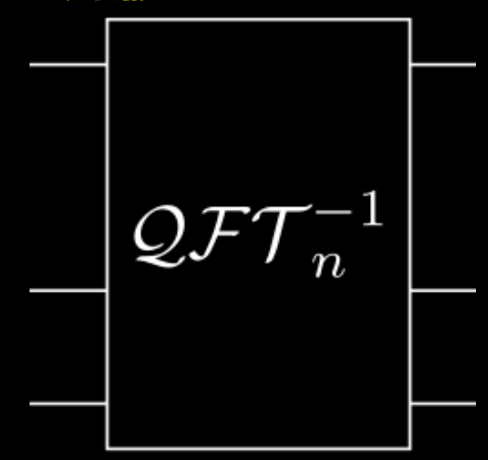
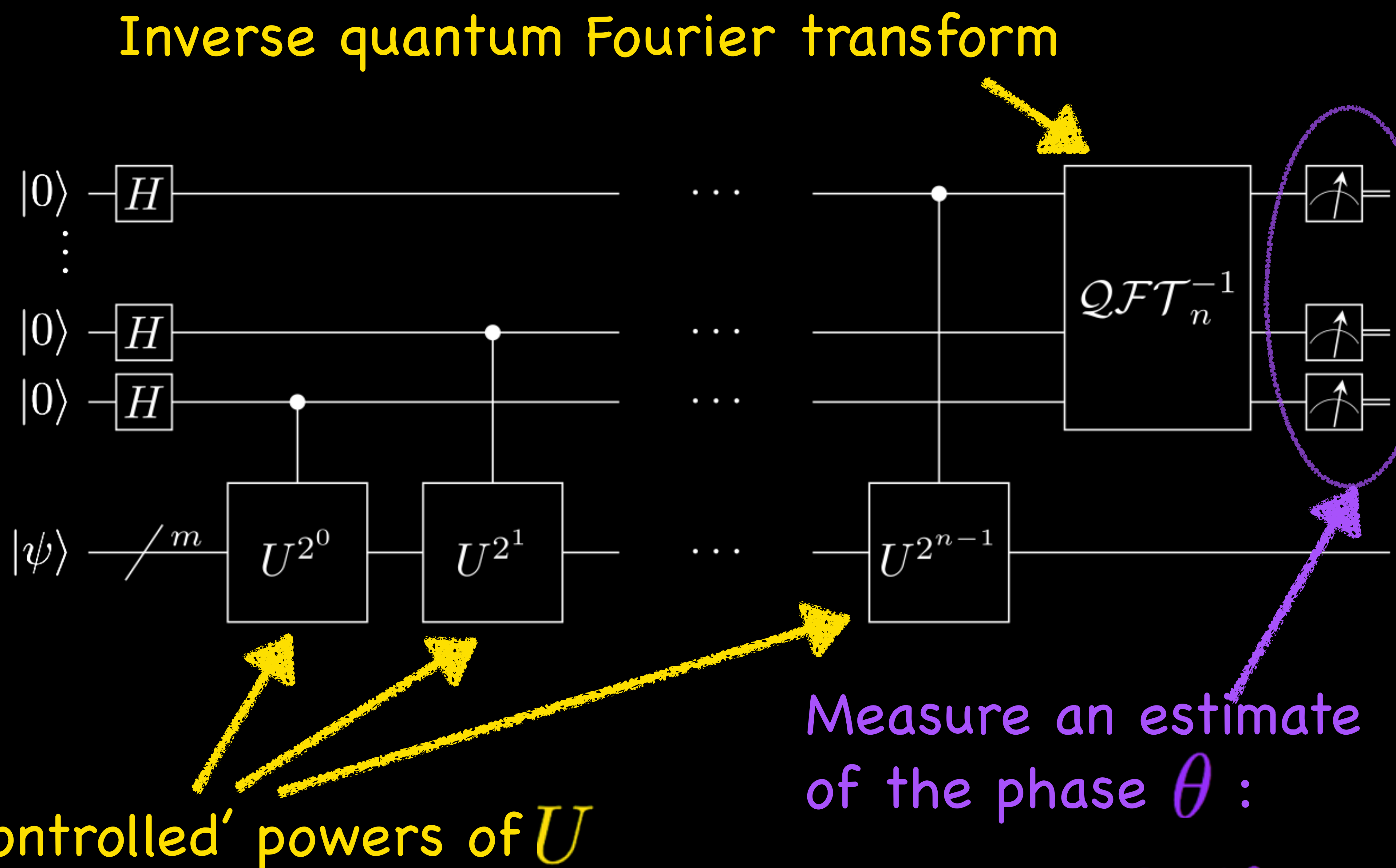


Quantum Fourier transform (QFT)

- Can be seen as a generalisation of the Hadamard gate
- Formally:

$$QFT_N|x\rangle = \frac{1}{\sqrt{N}} \sum_{y \in \{0,\ldots,N-1\}} e^{\frac{2\pi i x y}{N}} |y\rangle$$

Phase Estimation Circuit



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But a quantum computer can solve it quickly!

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22701801293785014193580405120204586741061235962766583907094021879215171483119139894870133091111044901683400949483846818299518041763507948922590774925466088171879259465921026597046700449819899096862039460017743094473811056991294128542891880855362707407670722593737772666973440977361243336397308051763091506836310795312607239520365290032105848839507981452307299417185715796297454995023505316040919859193718023307414880446217922800831766040938656344571034778553457121080530736394535923932651866030515041060966437313323672831539323500067937107541955437362433248361242525945868802353916766181532375855504886901432221349733

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22701801293785014193580405120204586741061235962766583907094021879215 1
71483119139894870133091111044901683400949483846818299518041763507948 9
22590774925466088171879259465921026597046700449819899096862039460017 7
43094473811056991294128542891880855362707407670722593737772666973440 9
77361243336397308051763091506836310795312607239520365290032105848839 5
07981452307299417185715796297454995023505316040919859193718023307414 8
80446217922800831766040938656344571034778553457121080530736394535923 9
32651866030515041060966437313323672831539323500067937107541955437362 4
3324836124252594586880235391676618153237585550488690143222134973 3

= ? x ?

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Best known classical algorithm : ~ 1 Billion Years

Shor's algorithm : ~ 100 Seconds*

*For a quantum computer running at ~1GHz.
Source: "A Compare between Shor's quantum factoring algorithm and General Number Field Sieve", Hamdi et al.

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3. Randomly choose $1<q<N-1$
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4. **Determine the order k of q modulo N**
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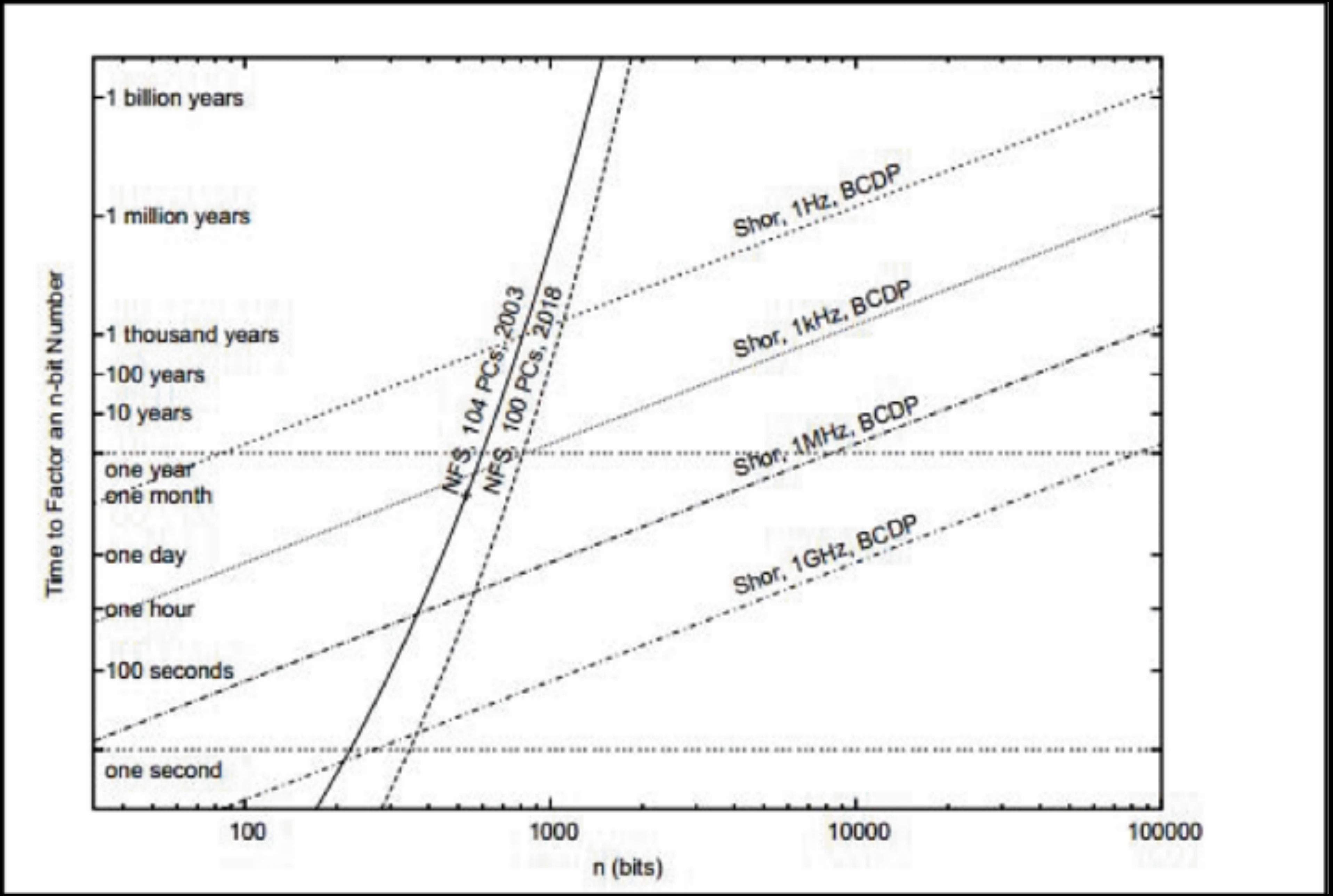
Fast Quantum algorithm using Phase Estimation

All other steps can be performed efficiently by a classical computer.

Given an *n*-bit integer:

Classical **number field sieve :** $O(2^{n^{1/3}})$

Shor's algorithm : $O(n^3)$



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- When H doesn't change over time, the solution to this equation is

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Hamiltonian Simulation:

Given a Hamiltonian H, construct a quantum circuit that approximates e^{-iHt}

There are a number of quantum algorithms that can do this efficiently for certain types of Hamiltonian

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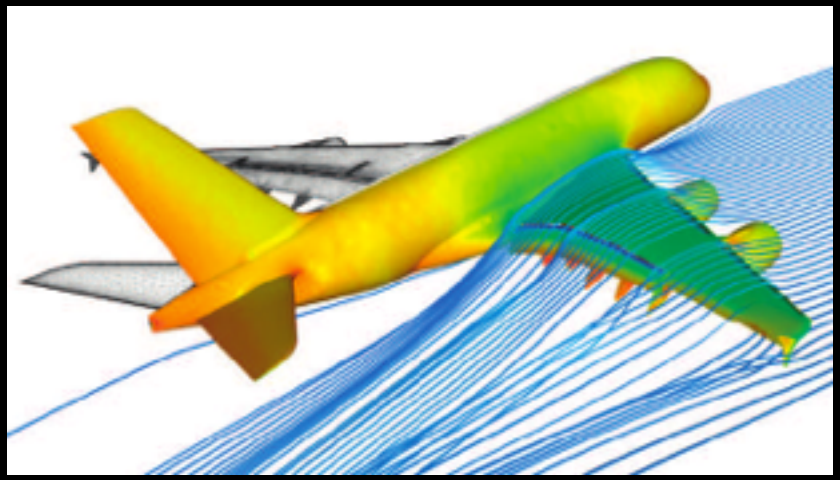

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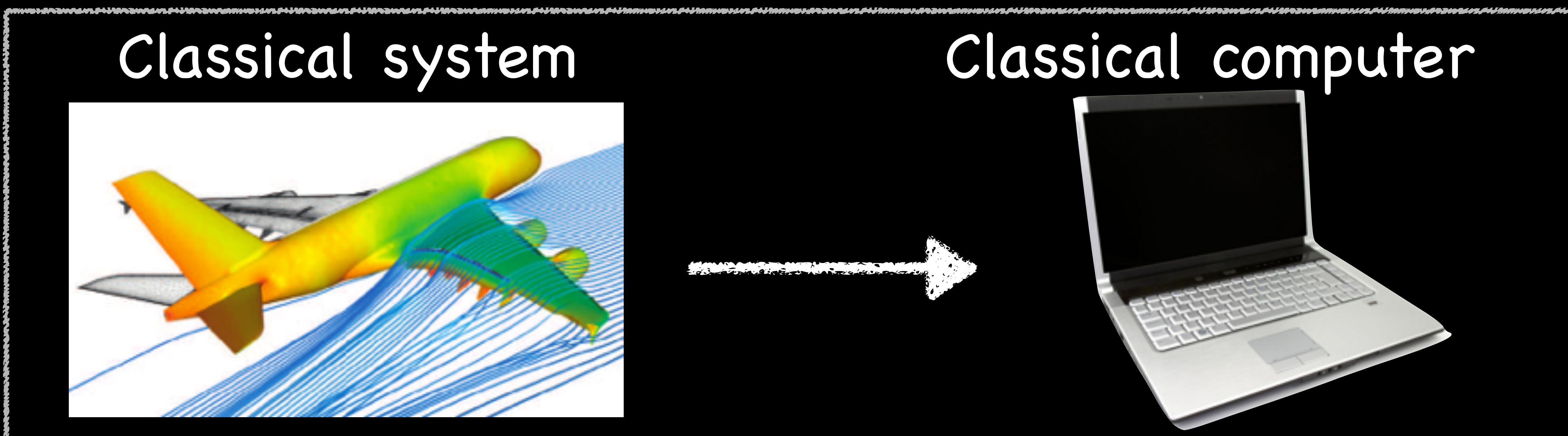

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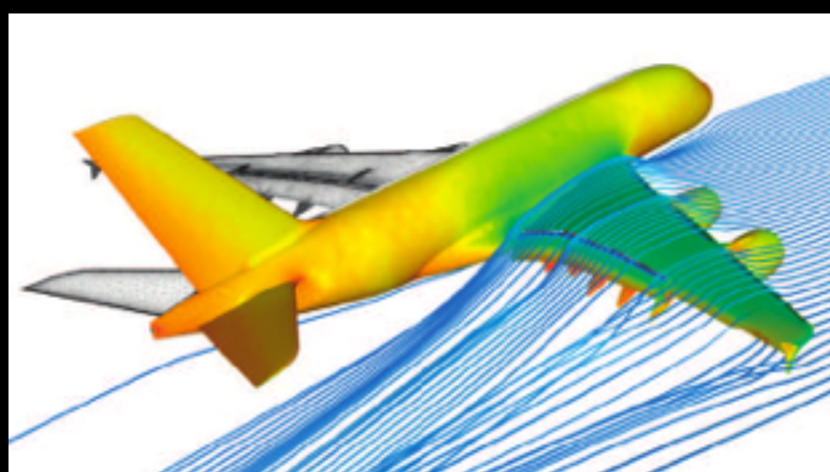
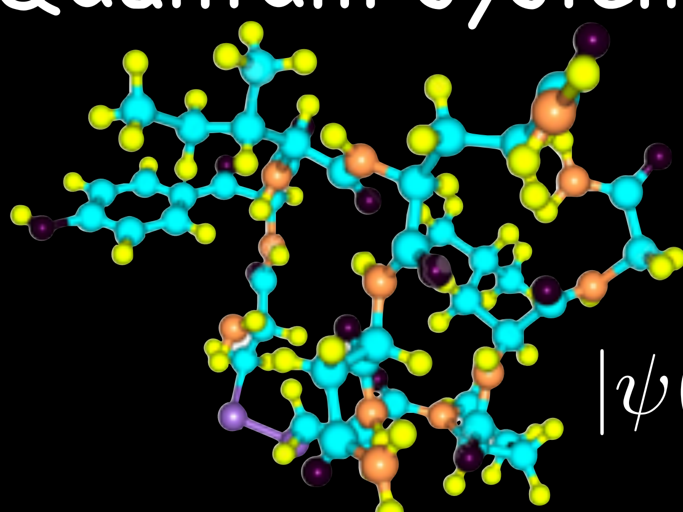


Classical computer

Quantum system



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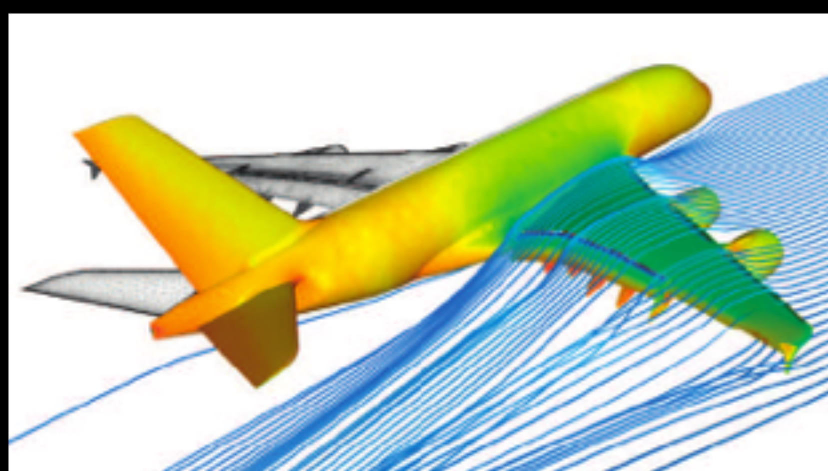
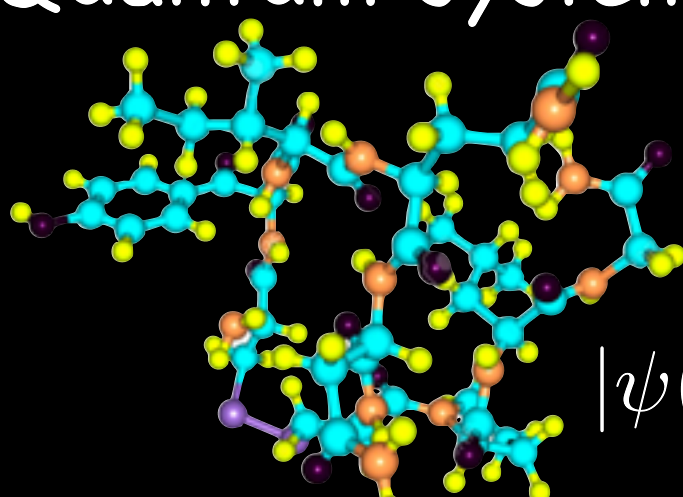
- If we want to simulate them on a computer, we're going to need a quantum one

Classical system

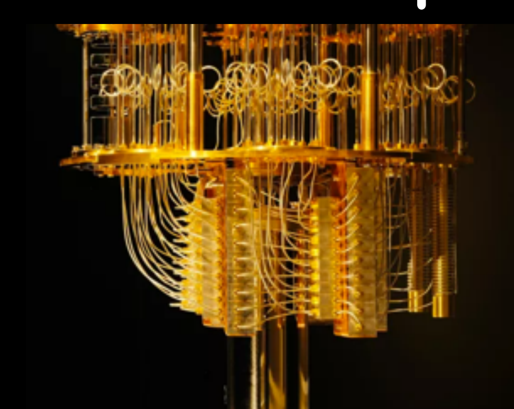


Classical computer

Quantum system



Quantum computer



$$|\psi(t)\rangle = e^{-iHt}|\psi(0)\rangle$$

HHL

Named after Harrow, Hassidim, and Lloyd, who invented it in 2008

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HHL '*solves*' this problem in logarithmic time

HHL

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Input: $|b\rangle, A$

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So long as we can prepare $|b\rangle$, and only need to know global properties of $|x\rangle$, this is useful.

Rough Outline

Rough Outline

Input: A
$|b\rangle$

Rough Outline

Input: A → $U = e^{iA}$

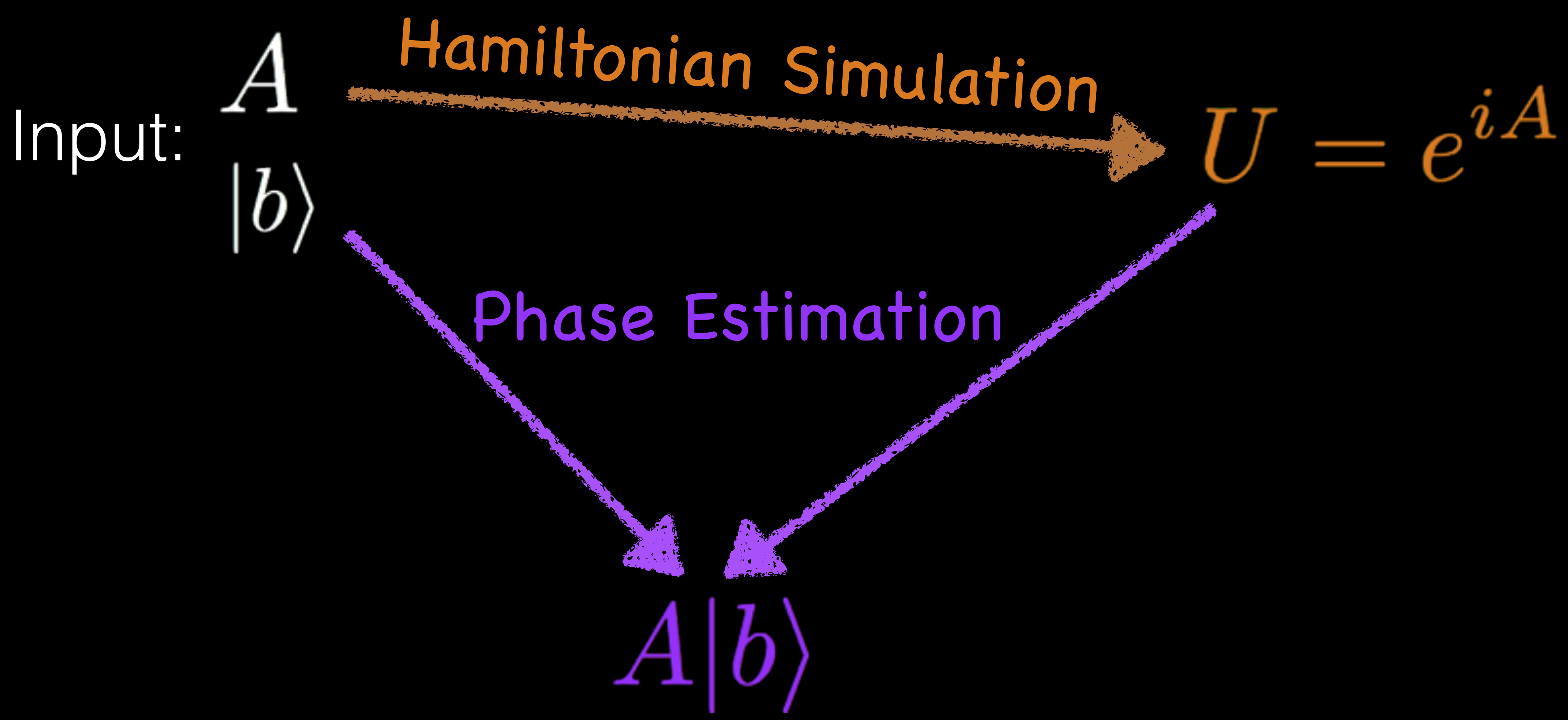
$|b\rangle$

Rough Outline

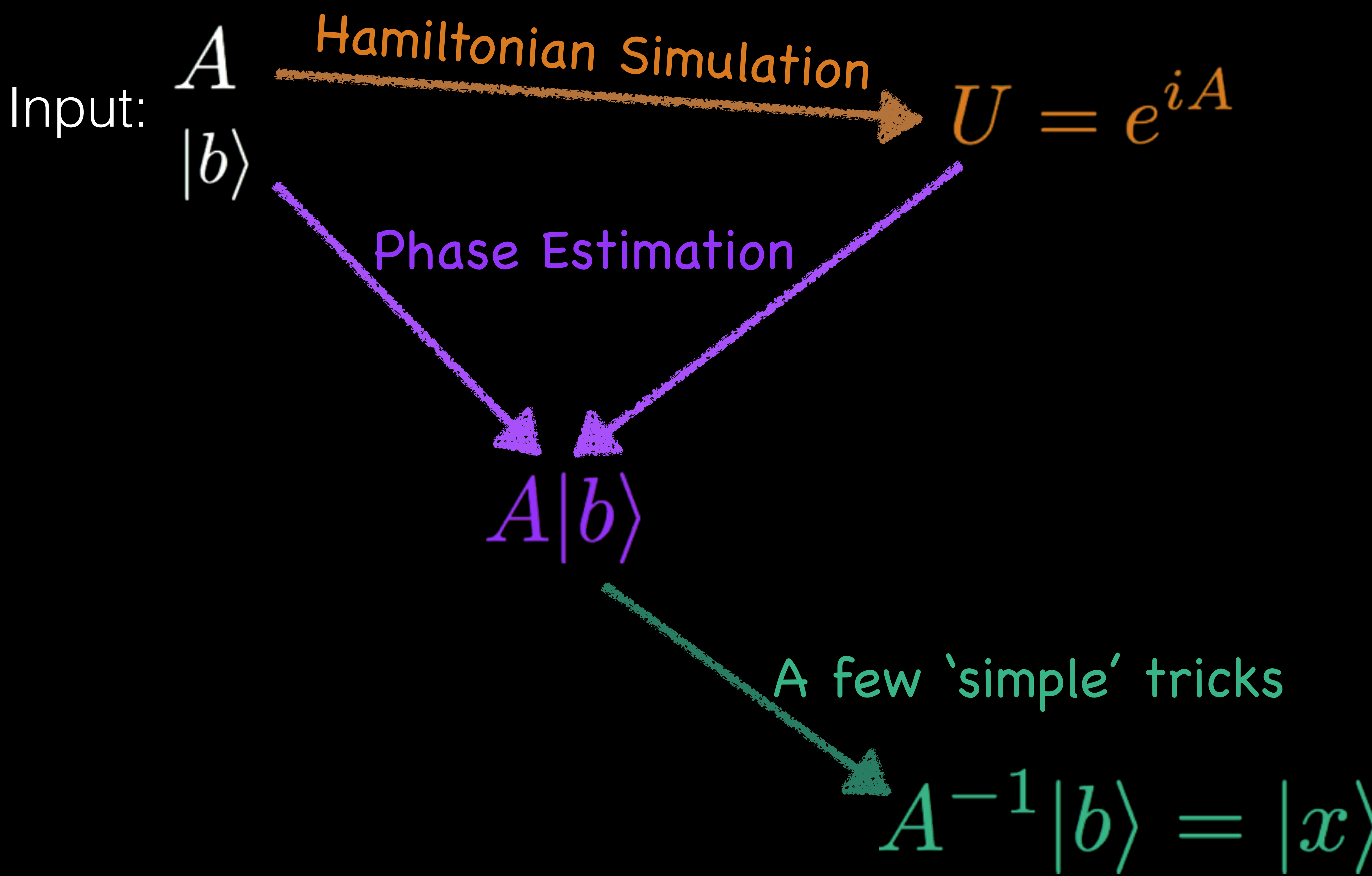
Input: A $|b\rangle$

Hamiltonian Simulation → $U = e^{iA}$

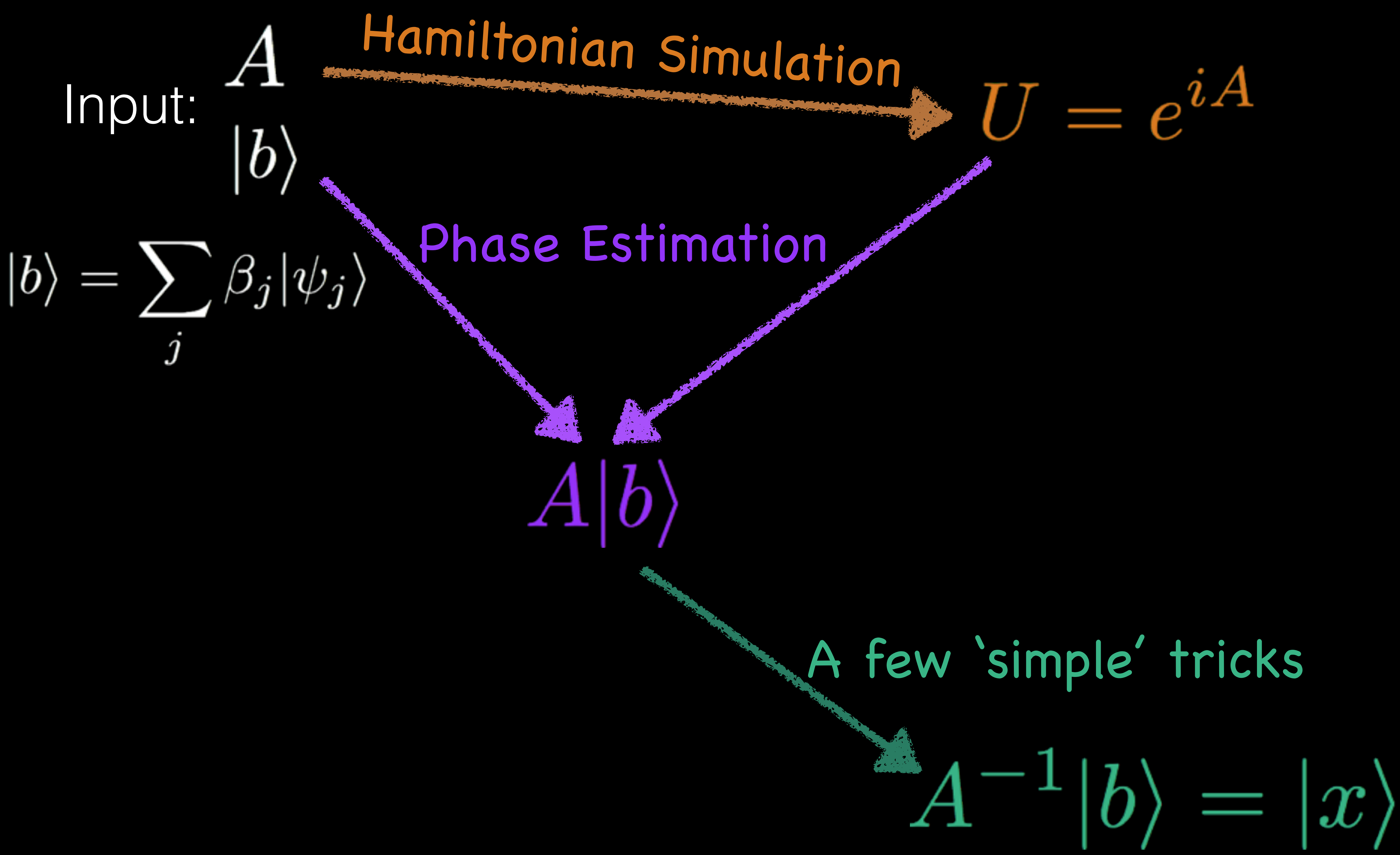
Rough Outline



Rough Outline



Rough Outline



Rough Outline

Input: A

$|b\rangle$

Hamiltonian Simulation

$U = e^{iA}$

$$|b\rangle = \sum_j \beta_j |\psi_j\rangle$$

Phase Estimation

$$A|b\rangle = \sum_j \beta_j |\lambda_j\rangle|\psi_j\rangle$$

A few ‘simple’ tricks

$$A^{-1}|b\rangle = |x\rangle$$

Rough Outline

Input: A, $|b\rangle$

$|b\rangle = \sum_j \beta_j |\psi_j\rangle$

Hamiltonian Simulation → $U = e^{iA}$

Phase Estimation → $A|b\rangle = \sum_j \beta_j |\lambda_j\rangle |\psi_j\rangle$

A few 'simple' tricks → $A^{-1}|b\rangle = |x\rangle$

$$\sum_j \beta_j |\psi_j\rangle \left(\sqrt{1 - \frac{1}{\lambda_j}} |0\rangle + \frac{1}{\lambda_j} |1\rangle \right)$$

Rough Outline

Input: A, $|b\rangle$

A → Hamiltonian Simulation → $U = e^{iA}$

$$|b\rangle = \sum_j \beta_j |\psi_j\rangle$$

Phase Estimation

$$A|b\rangle = \sum_j \beta_j |\lambda_j\rangle |\psi_j\rangle$$

A few 'simple' tricks

$$A^{-1}|b\rangle = |x\rangle$$

$$\sum_j \beta_j |\psi_j\rangle \left(\sqrt{1 - \frac{1}{\lambda_j}} |0\rangle + \frac{1}{\lambda_j} |1\rangle \right)$$

$$\sum_j \frac{\beta_j}{\lambda_j} |\psi_j\rangle = A^{-1}|b\rangle = |x\rangle$$

Applications

Applications

- Solving Systems of Differential Equations
  - E.g. Finite Element Method (FEM)
- Data fitting
- Various tasks in machine learning
  - E.g. clustering, support-vector machines, principal component analysis

Run-time

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For a system of ***n*** equations:

Run-time

For a system of ***n*** equations:

Classical : polynomial in ***n***

Run-time

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Quantum : logarithmic in ***n***

Run-time

For a system of ***n*** equations:

Classical : polynomial in ***n*** $\tilde{O}(n^3)$

Quantum : logarithmic in ***n*** $O(\kappa s(\log n)/\epsilon)$

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Condition number

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Sparsity

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For a system of ***n*** equations:

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Sparsity

Accuracy

Summary

- We saw some of the most frequently used quantum algorithms and sub-routines
    - Grover’s Search
    - Phase Estimation
    - Factoring
    - Hamiltonian Simulation
    - Matrix Inversion
- There are a lot more quantum algorithms, this is just a taster!
- Finding real-world applications is an ongoing challenge