Cheat Sheet

- $|0\rangle := \begin{pmatrix}1\\0\end{pmatrix}$, $|1\rangle := \begin{pmatrix}0\\1\end{pmatrix}$ $\rightarrow \langle 0|1\rangle = \langle 1|0\rangle = 0$, $\langle 0|0\rangle = \langle 1|1\rangle = 1$

  $|+\rangle := \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$, $|-\rangle := \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$

  $\rightarrow \langle +|-\rangle = \langle -|+\rangle = 0$, $\langle +|+\rangle = \langle -|-\rangle = 1$, $\langle 0|+\rangle = \langle 1|+\rangle = \langle 0|-\rangle = -\langle 1|-\rangle = \frac{1}{\sqrt{2}}$

- Born rule: given $|\psi\rangle$, the probability to measure $|x\rangle$ is given by $P(x) = |\langle x|\psi\rangle|^2$

- basic quantum gates:

  - single-qubit: * Pauli-matrices: $\sigma_x = \begin{pmatrix}0&1\\1&0\end{pmatrix}$, $\sigma_z = \begin{pmatrix}1&0\\0&-1\end{pmatrix}$, $\sigma_y = \begin{pmatrix}0&-i\\i&0\end{pmatrix} = i\sigma_x\sigma_z$

    ↳ bit flip ↳ phase flip ↳ bit & phase flip

    $\rightarrow \sigma_i^2 = \mathbb{1} = \begin{pmatrix}1&0\\0&1\end{pmatrix}$ → does nothing

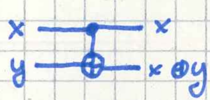
    * Hadamard gate: $H = \frac{1}{\sqrt{2}}\begin{pmatrix}1&1\\1&-1\end{pmatrix} = |+\rangle\langle 0| + |-\rangle\langle 1|$

    $\rightarrow H^{\otimes n} \cdot |x\rangle = \frac{1}{\sqrt{2^n}} \sum_{k \in \{0,1\}^n} (-1)^{\langle k|x\rangle} \cdot |k\rangle$

    * phase shift gates: $R_\phi = \begin{pmatrix}1&0\\0&e^{i\phi}\end{pmatrix}$

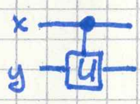
    e.g. $\frac{\pi}{8}$-gate: $T = \begin{pmatrix}1&0\\0&e^{i\pi/4}\end{pmatrix}$, phase gate: $S = \begin{pmatrix}1&0\\0&i\end{pmatrix}$

  - two-qubit: * $CNOT = \begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix}$:



    ↳ applies σ_x iff $x=1$, else nothing happens

    * controlled-U gate $cU = \begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&&\\0&0&&U\end{pmatrix}$



    ↳ applies U iff $x=1$, else nothing happens

    * $SWAP = \begin{pmatrix}1&0&0&0\\0&0&1&0\\0&1&0&0\\0&0&0&1\end{pmatrix}$ → swaps the states of two qubits

⇒ a simple set of universal quantum gates is $\{H, T, CNOT\}$

↳ any quantum operation can be approximated by a sequence of gates from this set