Quantum teleport example

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September 9, 2019

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Outline

Quantum Teleportation

Introduction

Quick recap'

Computational basis states

$$|0\rangle = \begin{bmatrix}1\\0\end{bmatrix} \qquad |1\rangle = \begin{bmatrix}0\\1\end{bmatrix}$$

General qubit state

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

Hadamard basis states

$$|+\rangle = H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \qquad |-\rangle = H|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

Setting up the initial states

Alice has an unknown state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ that she wants to transmit to Bob.

Alice and Bob share a pair of entangled qubits $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$.

Let's write the full joint qubit state:

$$|\psi_{\text{full joint}}\rangle = \frac{1}{\sqrt{2}}[\alpha|0\rangle(|00\rangle + |11\rangle) + \beta|0\rangle(|00\rangle + |11\rangle)]$$

Change to the Hadamard basis

Alice changes the basis of her unknown state to the Hadamard basis

$$\begin{aligned} H|\psi\rangle &= H(\alpha|0\rangle + \beta|1\rangle) \\ &= \frac{1}{\sqrt{2}}[\alpha(|0\rangle + |1\rangle) + \beta(|0\rangle - |1\rangle)] \end{aligned}$$

Now rewrite Alice's and Bob's joint qubit state (after the Hadamard transform):

$$|\psi_{\text{full joint}}\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} \frac{\alpha}{\sqrt{2}}(|0\rangle + |1\rangle)(|00\rangle + |11\rangle) \\ +\frac{\beta}{\sqrt{2}}(|0\rangle - |1\rangle)(|00\rangle + |11\rangle) \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} \alpha(|000\rangle + |011\rangle + |100\rangle + |111\rangle) \\ +\beta(|001\rangle + |010\rangle - |101\rangle + |110\rangle) \end{bmatrix}$$

Change to the Hadamard basis (contd.)

$$|\psi_{\text{full joint}}\rangle = \frac{1}{2}\begin{bmatrix} |00\rangle(\alpha|0\rangle + \beta|1\rangle) \\ |01\rangle(\alpha|1\rangle + \beta|0\rangle) \\ |10\rangle(\alpha|0\rangle - \beta|1\rangle) \\ |11\rangle(\alpha|1\rangle - \beta|0\rangle) \end{bmatrix}$$

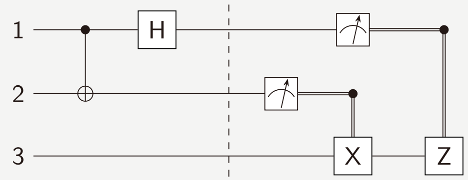
- Note that the state of the third qubit is very similar to the Alice's $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$!
- If Alice measures her two qubits, Bob's qubit will collapse to one of the above states.
- Bob only needs to know the two measurements results and apply some correction to his qubit if required and... Voilà!

Programming quantum teleportation

Quantum circuit

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$



Recall, at the dashed line, the state of our qubits is given by:

$$|\psi_{\text{full joint}}\rangle = \frac{1}{2}\begin{bmatrix} |00\rangle(\alpha|0\rangle + \beta|1\rangle) \\ |01\rangle(\alpha|1\rangle + \beta|0\rangle) \\ |10\rangle(\alpha|0\rangle - \beta|1\rangle) \\ |11\rangle(\alpha|1\rangle - \beta|0\rangle) \end{bmatrix}$$

Implementation I

```python
from projectq import MainEngine
from projectq.ops import CNOT, H, Measure, X, Rz, Z
from projectq.meta import Dagger

def create_state(eng, qb):
    H | qb
    Rz(1.21) | qb


eng = MainEngine()

# ...
```


Implementation II

```python
# ...

# Create a Bell pair
b1, b2 = eng.allocate_qureg(2)
H | b1
CNOT | (b1, b2)

# Create state to send
psi = eng.allocate_qubit()
create_state(eng, psi)

# ...
```

Implementation III

```
# ...

# Entangle psi with Alice's other qubit
CNOT | (psi, b1)

# Alice measures her two qubits
H | psi
Measure | psi
Measure | b1
msg_to_bob = [int(psi), int(b1)]

# ...
```


Implementation IV

```
# ...
# Bob received the message and applies the corrections (if needed)
if msg_to_bob[0]:
    Z | b2
if msg_to_bob[1]:
    X | b2

# Test if teleport successful: uncompute the state
with Dagger(eng):
    create_state(eng, b2)
del b2  # will throw if not in classical state
eng.flush()
```


Whole code

```
from projectq import MainEngine
from projectq.ops import (CNOT, H, Measure,
                          X, Rz, Z)
from projectq.meta import Dagger

def create_state(eng, qb):
    H | qb
    Rz(1.21) | qb


eng = MainEngine()

# Create a Bell pair
b1, b2 = eng.allocate_qureg(2)
H | b1
CNOT | (b1, b2)

# Create state to send
psi = eng.allocate_qubit()
create_state(eng, psi)
```

```
# Entangle with Alice's
CNOT | (psi, b1)

# Alice measures her two qubits
H | psi
Measure | psi
Measure | b1
msg_to_bob = [int(psi), int(b1)]

# Bob applies the corrections (if needed)
if msg_to_bob[0]:
    Z | b2
if msg_to_bob[1]:
    X | b2

# Test if teleport successful: uncompute
with Dagger(eng):
    create_state(eng, b2)
del b2  # will throw if not classical state
eng.flush()
```


Thank you for your attention

Any questions?

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